Nonlinear Regge Trajectories in Theory and Practice

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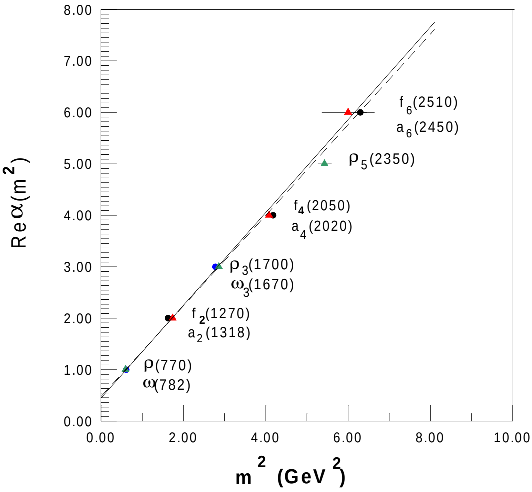
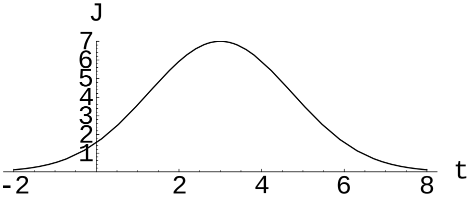
Spectroscopy



(Desgrolard-Giffon-Martynov-Predazzi,2000)

Theory

(pre QCD) $\lambda\phi^3$, $\lambda\phi^4$ (Lee-Sawyer, 1962)



$$\alpha(t) = const + \lambda^2 K(t)$$

$$\alpha(t)|_{t\to-\infty} \to \begin{cases} -1, & \phi^3 \\ 0, & \phi^4 \end{cases}$$

$$\alpha_P(t) = 1 + \gamma(\sqrt{t_0} - \sqrt{t_0 - t}) \to -\infty, \quad t \to -\infty$$

(Fiore-Jenkovszky-Paccanoni-Prokudin, 2003)

$$\alpha(t) = c \ln(b - at), \quad c \sim g^2 \quad (\text{``}\textit{vector gluon}''\textit{coupling})$$

(Coon-Suura, 1974)

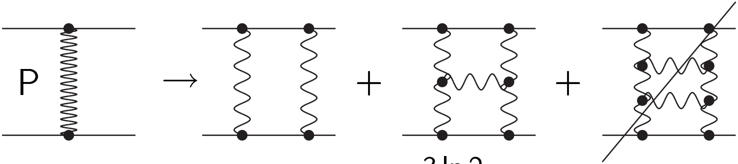
$$\alpha(t) = a + bt + b\sqrt{(t_0 - t)(t_0^* - t)} \to \begin{cases} 2bt, \ t \to +\infty \\ -1, \ t \to -\infty \end{cases}$$

(Collins-Kearney, 1984)

+ many others

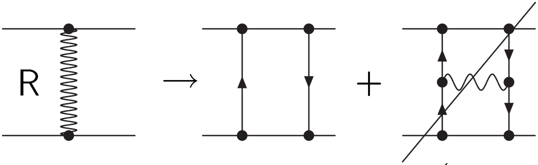
QCD

high(-t)



$$\alpha_P(t) \rightarrow 1 + \frac{3\ln 2}{\pi^2} g^2(t) + O(g^{10/3})$$

(Kirschner-Lipatov,1990)



$$\alpha_R(t) \rightarrow 0 + O\left(\sqrt{g^2(t)}\right)$$

(Kwiecinski,1982)

low t:

$$\alpha_P(0) = \frac{3\ln 2}{\pi^2} g^2 \left(1 - \frac{5}{\pi^2} g^2\right)$$

(Fadin-Lipatov; Camici-Ciafaloni, 1998)

Renormalization group invariance of a physical quantity P:

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(g^2) \frac{\partial}{\partial g^2}\right] P = 0$$

$$P = \alpha(t) \to \alpha(t) = \Phi\left[\frac{t}{\mu^2} e^{K(g^2)}\right]$$

$$\frac{d}{dg^2} K(g^2) = \frac{1}{\beta(g^2)}, \quad \beta(g^2) = -\beta_0 g^4 - \beta_1 g^6 - \dots$$

$$16\pi^2 \beta_0 = 11 - 2n_f/3, \quad 128\pi^4 \beta_1 = 51 - 19n_f/3$$

$$K(g^2) \approx \frac{1}{\beta_0 g^2} + \frac{\beta_1}{\beta_0^2} \ln \frac{1}{g^2} + \dots$$

Simple consequences of RG invariance:

1) $\alpha(0) = \Phi(0)$, i.e. the intercept does not depend on g^2

2) If $\alpha(t)$ is analytic at $t = 0$ then

$$\alpha(t) = \Phi(0) + \Phi'(0)\frac{t}{\mu^2}e^{K(g^2)} + ... = \alpha(0) + \alpha'(0)t + ...$$

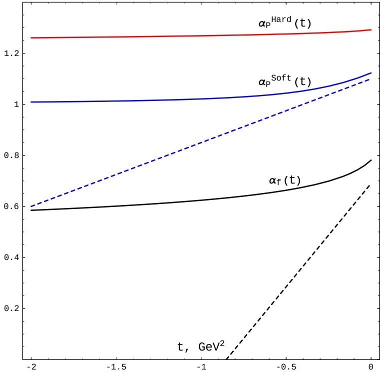
$$\alpha'(0)\Big|_{g^2 \to 0} \sim \left(\frac{1}{g^2}\right)^{\frac{\beta_1}{\beta_0^2}} e^{\frac{1}{\beta_0 g^2}}$$

$$\alpha(t) = f[g^2(t)], \quad \alpha(0) = f[g^2(0)] = \Phi\left[e^{K(g^2(0))}\right]$$

$$K(g^2(0)) = -\infty \quad \left(K(g^2(t) - K(g^2) = \ln\left(\frac{-t}{\mu^2}\right)\right)$$

$g^2(0) < \infty$ (Shirkov-Solovtsov, 1997)

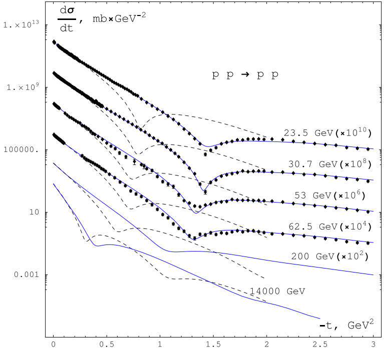
Practice



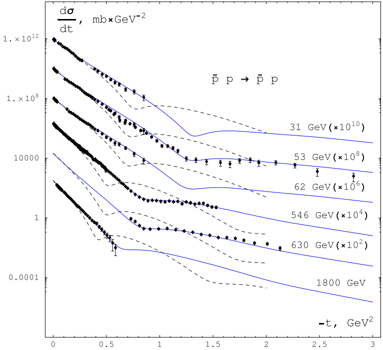
$$\alpha_P^{Soft}(t) = 1 + p_1 \left[1 - p_2\, t \left(arctg(p_3 - p_2\, t) - \frac{\pi}{2}\right)\right]$$

$$\alpha_P^{Hard}(t) = 1 + \frac{1}{A_H + [\frac{3\ln 2}{\pi^2} g_s^2(\sqrt{-t + c_H})]^{-1}}$$

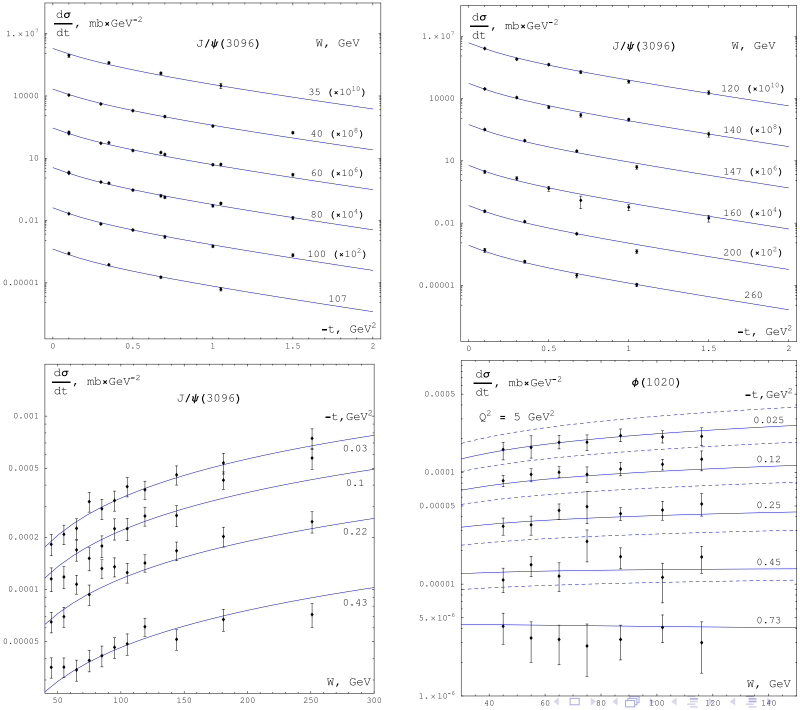
$$\alpha_R(t) = \left(\frac{2}{3\pi^2} g_s^2(\sqrt{-t + c_R})\right)^{1/2}, \quad g_s^2(\mu) \equiv \frac{(4\pi)^2}{11 - \frac{2}{3} n_f} \left(\frac{1}{\ln \frac{\mu^2}{\Lambda^2}} + \frac{1}{1 - \frac{\mu^2}{\Lambda^2}}\right)$$



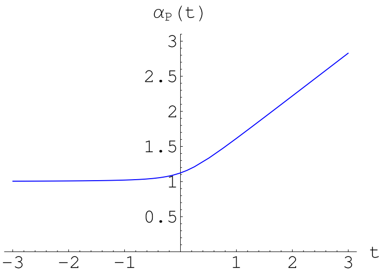
dσ/dt, mb×GeV^-2
p p → p p
23.5 GeV (×10^10)
30.7 GeV (×10^8)
53 GeV (×10^6)
62.5 GeV (×10^4)
200 GeV (×10^2)
14000 GeV
-t, GeV^2

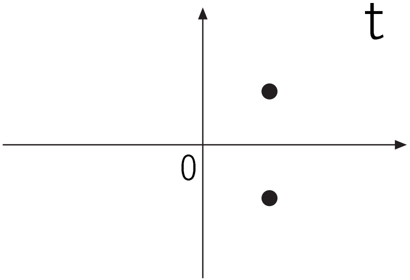


dσ/dt, mb×GeV^-2
p̄ p → p̄ p
31 GeV (×10^10)
53 GeV (×10^8)
62 GeV (×10^6)
546 GeV (×10^4)
630 GeV (×10^2)
1800 GeV
-t, GeV^2


dσ/dt, mb×GeV^{-2}
J/ψ(3096)
W, GeV
35 (×10^{10})
40 (×10^{8})
60 (×10^{6})
80 (×10^{4})
100 (×10^{2})
107
−t, GeV^{2}
1.×10^{7}
10000
10
0.01
0.00001
0
0.5
1
1.5
2
dσ/dt, mb×GeV^{-2}
J/ψ(3096)
W, GeV
120 (×10^{10})
140 (×10^{8})
147 (×10^{6})
160 (×10^{4})
200 (×10^{2})
260
−t, GeV^{2}
dσ/dt, mb×GeV^{-2}
J/ψ(3096)
−t, GeV^{2}
0.03
0.1
0.22
0.43
0.001
0.0005
0.0002
0.0001
0.00005
W, GeV
50
100
150
200
250
300
dσ/dt, mb×GeV^{-2}
φ(1020)
−t, GeV^{2}
Q^2 = 5 GeV^{2}
0.025
0.12
0.25
0.45
0.73
0.0005
0.0001
0.00005
0.00001
5.×10^{-6}
1.×10^{-6}
W, GeV
40
60
80
100
120
140

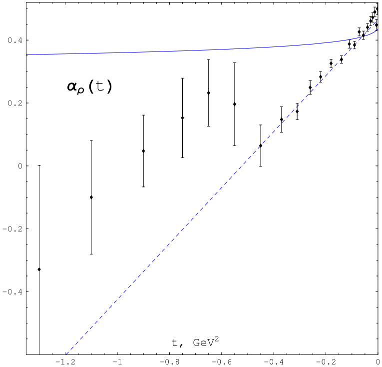
Problems





Microcausality is under threat? (V.P.-Samokhin, 1990)

Problems



(Brodsky-Tang-Thorn, 1993; Kaidalov, 2006)

π-trajectory?

Heavy quarks?

(Interim) conclusions:

1. Analyticity of RT in t seems to imply their singular behavior at $g^2 \sim 0$;

2. QCD (partonlike) asymptotics at high $(-t)$ and "stringy" asymptotics at high t imply complex singularities of RT in the t-plane. Probable clash with microcausality.

3. QCD behavior at high $(-t)$ does not match with monotony of RT in t for ρ, π,..., heavy quarkonia.