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Evolution equation for the unintegrated gluon distribution

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Abstract

We obtain the evolution equation for the unintegrated gluon distribution inside gluon. Our framework is based on the Bethe-Salpeter equation for the Green functions in the pure gluonic case.

1 Introduction

The aim of this paper is to construct the exact evolution equation for the unintegrated gluon distribution inside gluon (gUPDF). A lot of papers are devoted to the unintegrated parton distribution functions in hadrons (hUPDF) in different forms. Most of authors consider specific ”formalism” to obtain UPDF from integrated functions. One of the methods is based on the separation of virtual and real contributions in the usual DGLAP equation [1]-[5] and on the angular ordering (see [6] and reference therein). The most general result is the CCFM evolution equation, which reduces to the leading order DGLAP formalism at moderate x and to the BFKL one at small x. In the above cases the additional ”hard” scale μ is introduced by hand. Finally authors obtain unintegrated function that depends on two ”hard” scales: μ and $k_\perp$, which are related in a complicated way.

In our method we use the exact Bethe-Salpeter equation for the gUPDF. In our direct method we do not need to introduce the hard scale by hand, since it arises naturally from invariants. In this paper we present only the equation and the application of gUPDFs to the semi-inclusive central production process.

2 Calculations

2.1 Helicity method and 3-gluon vertex

Now we introduce usual 3-gluon amplitude to calculate the kernel of the equation for unintegrated distributions in the leading approximation. Notations for all the momenta are clear from the Fig. 1.

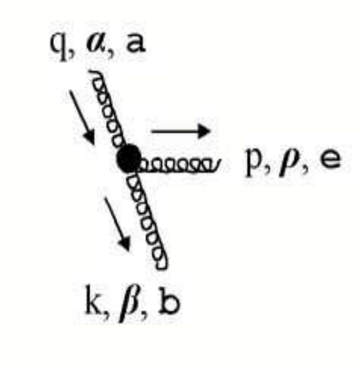

Figure 1: 3-gluon vertex with all the momenta and indices.

In the tensor form 3-gluon vertex can be written as usual:

$$M^{\mathrm{abe}}_{\alpha\beta\rho}(k,q) = g_s f^{\mathrm{abe}} \left[(p+q)_\beta \, g_{\rho\alpha} - (q+k)_\rho \, g_{\alpha\beta} - (p-k)_\alpha \, g_{\rho\beta} \right] \tag{1}$$

Here $g_s = \sqrt{4\pi\alpha_s}$, α_s is the strong coupling, f^{abe} is the structure constant of the SU(3) group, $p^2 = (q-k)^2 = 0$ (real gluon), $k^2 \leq q^2 \leq 0$.

Let us introduce polarization vectors for every gluon. All the gluons are considered in the light-like gauge with the axial vector n, $n^2 = 0$. We have the following expressions for polarization vectors:

$$\epsilon^\perp_\mu = N^{-1} \frac{\varepsilon_{\mu\nu\rho\sigma} q^\nu n^\rho k^\sigma}{pn} \; , \tag{2}$$

$$\epsilon^\parallel_p = N^{-1} \left(q - \frac{qn}{pn} p - \frac{qp}{pn} n \right) \; , \tag{3}$$

$$\epsilon^\parallel_q = N^{-1} \left(q - \frac{qn}{pn} p - \frac{kq\, qn - q^2 kn}{pn\; qn} n \right) \; , \tag{4}$$

$$\epsilon^\parallel_k = N^{-1} \left(k - \frac{kn}{pn} p + \frac{kq\, kn - k^2 qn}{pn\; kn} n \right) \; , \tag{5}$$

$$N = \sqrt{\frac{q^2 kn - k^2 qn}{pn}} \; , \; kq = \frac{k^2 + q^2}{2} \; , \tag{6}$$

which satisfy conditions:

$$r\varepsilon_r^{\parallel} = r\varepsilon^{\perp} = 0 \ , \tag{7}$$

$$n\varepsilon_r^{\parallel} = n\varepsilon^{\perp} = 0 \ , \tag{8}$$

$$\varepsilon_r^{\pm} = \frac{1}{\sqrt{2}}\left(\varepsilon_r^{\parallel} \pm \imath\, \varepsilon^{\perp}\right) \ , \tag{9}$$

$$\varepsilon^{+}_{r,\mu}\varepsilon^{-}_{r,\nu} + \varepsilon^{-}_{r,\mu}\varepsilon^{+}_{r,\nu} = -g_{\mu\nu} + \frac{r_\mu n_\nu + r_\nu n_\mu}{rn} - \frac{r^2}{(rn)^2} n_\mu n_\nu \ , \tag{10}$$

$$\varepsilon_r^{-}\varepsilon_r^{+} = -1 \ , \ \varepsilon_r^{\pm}\varepsilon_r^{\pm} = 0 \ , \tag{11}$$

where $r = q, k, p$. For particles with nonzero virtualities we have additional polarizations, but we can consider only $\varepsilon^{\pm}$ polarization vectors, since helicity amplitudes give the main contribution to the kernel, which is simply the amplitude squared.

The expression for the helicity amplitude is the following:

$$\begin{aligned} & M^{\mathrm{abe}}_{\lambda_q\lambda_k\lambda_p}(k^2, q^2, z) = \\ & = g_s f^{\mathrm{abe}} \frac{N}{\sqrt{2}} \left[\frac{z}{1-z}(\lambda_q\lambda_p - 1) - z(\lambda_k\lambda_p - 1) + (\lambda_q\lambda_k - 1)\right] , \end{aligned} \tag{12}$$

$$N^2 = \frac{q^2(1-z) - k^2}{z} \ , \tag{13}$$

$$z = \frac{pn}{qn} \ , \ \lambda_r = \pm 1 \ , \tag{14}$$

2.2 Evolution equation

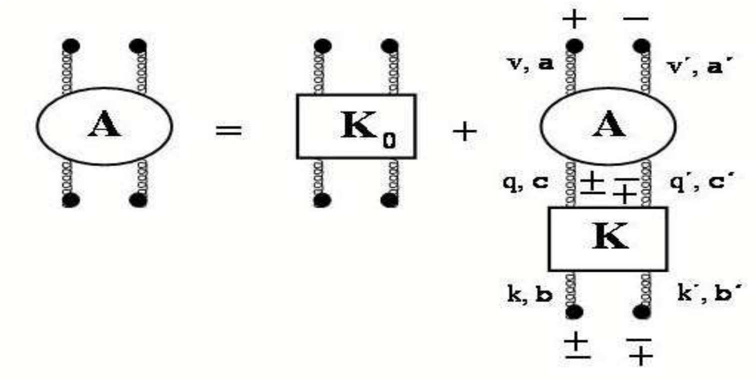

Figure 2: Evolution equation for unintegrated gluon distribution inside a gluon.

Now we can write the kernel of the evolution equation in the leading order:

$$\begin{aligned} g_s^2 f^{\mathrm{cbe}} f^{\mathrm{c'b'e}} K_{\lambda_q\,-\lambda_q,\,\lambda_k\,-\lambda_k}(k^2, q^2, z) &= M^{\mathrm{cbe}}_{\lambda_q\lambda_k\,+}(k^2, q^2, z) M^{\mathrm{c'b'e}}_{-\lambda_q\,-\lambda_k\,-}(k^2, q^2, z) + \\ &+ M^{\mathrm{cbe}}_{\lambda_q\lambda_k\,-}(k^2, q^2, z) M^{\mathrm{c'b'e}}_{-\lambda_q\,-\lambda_k\,+}(k^2, q^2, z) \ , \end{aligned} \tag{15}$$

$$\begin{aligned} K_{11} = K_{22} = K_{+-,\,-+} &= K_{-+,\,+-} = \\ &= \frac{2NN'}{(1-z)^2}\left(1 + (1-z)^4\right) \ , \end{aligned} \tag{16}$$

$$\begin{aligned} K_{12} = K_{21} = K_{+-,\,+-} &= K_{-+,\,-+} = \\ &= \frac{2NN'}{(1-z)^2} z^4 \ , \end{aligned} \tag{17}$$

Notations are clear from the Fig. 2.

We have for the evolution equation

$$A_{\mathrm{i}} \ = \ K_{0,\mathrm{i}} + K_{\mathrm{ij}} \otimes A_{\mathrm{j}} \ , \tag{18}$$

$$A_1 \equiv A^{\rm ab,a'b'}_{+-,\,+-},\; A_2 \equiv A^{\rm ab,a'b'}_{+-,\,-+}\,, \tag{19}$$

and $\otimes$ means

$$4\pi\alpha_s f^{\rm abe} f^{\rm a'b'e} \int \frac{d^4q}{(2\pi)^3 q^4}\delta((q-k)^2)\theta(qn-kn)\theta(vn-qn) \tag{20}$$

We can simplify the equation, if we take $N = N'$ and introduce new functions:

$$f_1^{\rm ab,\,a'b'} = A_1 + A_2\,, \tag{21}$$

$$f_2^{\rm ab,\,a'b'} = A_1 - A_2\,, \tag{22}$$

$$V_1 = K_{11} + K_{12} = \frac{4(q^2(1-z)-k^2)(1-z(1-z))^2}{z(1-z)^2}\,, \tag{23}$$

$$V_2 = K_{11} - K_{12} = \frac{4(q^2(1-z)-k^2)(1-z(1-2z))}{z(1-z)}\,, \tag{24}$$

and color operators:

$$f^{\rm abe} f^{\rm a'b'e} = \left(\sum_{\rm I}\lambda_{\rm I}\hat{\mathcal{P}}_{\rm I}\right)^{\rm aa',\,bb'}, \quad {\rm I} = 1, 8, \bar{8}, 10, \bar{10}, 27;$$

$$\lambda_{\rm I} = N_c, \frac{N_c}{2}, \frac{N_c}{2}, 0, 0, -1;\; {\rm C_I} = \frac{1}{64}, \frac{8}{64}, \frac{8}{64}, \frac{10}{64}, \frac{10}{64}, \frac{27}{64}; \tag{25}$$

$${\rm C_I}\lambda_{\rm I} = 0\,, \tag{26}$$

where $N_c = 3$, $\rm C_I$ are weights for corresponding cross-sections and $\hat{\mathcal{P}}_{\rm I}$ are color projection operators for irreducible tensor representations of the SU(3) group:

$$\hat{\mathcal{P}}_1^{\rm aa',\,bb'} = \frac{1}{N_c^2-1}\delta^{\rm aa'}\delta^{\rm bb'}\,, \tag{27}$$

$$\hat{\mathcal{P}}_8^{\rm aa',\,bb'} = \frac{N_c}{N_c^2-4} d^{\rm aa'e} d^{\rm bb'e}\,, \tag{28}$$

$$\hat{\mathcal{P}}_{\bar{8}}^{\rm aa',\,bb'} = \frac{1}{N_c} f^{\rm aa'e} f^{\rm bb'e}\,, \tag{29}$$

$$\hat{\mathcal{P}}_{27}^{\rm aa',\,bb'} = \frac{1}{N_c}\left(\delta^{\rm ab}\delta^{\rm a'b'} + \delta^{\rm ab'}\delta^{\rm ba'}\right) - \frac{N_c^2-2}{N_c(N_c^2-1)}\delta^{\rm aa'}\delta^{\rm bb'} + $$
$$+ \frac{1}{2}\left(d^{\rm abe}d^{\rm a'b'e} + d^{\rm ab'e}d^{\rm ba'e}\right) - \frac{N_c^2-8}{2(N_c^2-4)} d^{\rm aa'e}d^{\rm bb'e}\,, \tag{30}$$

$$\hat{\mathcal{P}}_{\rm I}^{\rm aa',\,cc'}\hat{\mathcal{P}}_{\rm J}^{\rm cc',\,bb'} = \delta_{\rm IJ}\hat{\mathcal{P}}_{\rm I}^{\rm aa',\,bb'}\,,\; \hat{\mathcal{P}}_{\rm J}^{\rm aa',\,aa'} = {\rm J}\,,\; \sum_{\rm I}\hat{\mathcal{P}}_{\rm I}^{\rm aa',\,bb'} = \delta^{\rm ab}\delta^{\rm a'b'}\,, \tag{31}$$

$d^{\rm abc}$ is the symmetric structure constant of the SU(3) group. Now we can write the diagonalized evolution equation:

$$f_{\rm i}^{\rm ab,\,a'b'}(v^2, k^2, k_\perp^2, \hat{x}) = f_{0,\rm i}^{\rm ab,\,a'b'}(v^2, k^2, k_\perp^2, \hat{x}) + 4\pi\alpha_s(k^2)\int\frac{d^4q}{(2\pi)^3q^4}\delta((q-k)^2)$$
$$\theta(qn-kn)\theta(vn-qn) \cdot V_{\rm i}(q^2, k^2, z) f_{\rm i}^{\rm ac,\,a'c'}(v^2, q^2, q_\perp^2, \frac{\hat{x}}{\hat{z}})\left(\sum_{\rm I}\lambda_{\rm I}\hat{\mathcal{P}}_{\rm I}\right)^{\rm cc',\,bb'}. \tag{32}$$

For $f_{\rm i}$ we have the following expansion:

$$f_{\rm i}^{\rm ab,\,a'b'} = \frac{1}{N_c^2-1}\sum_{\rm J} f_{\rm i,J}\hat{\mathcal{P}}_{\rm J}^{\rm aa',\,bb'} + {\rm o}(\alpha_{\rm s})\,, \tag{33}$$

and making the contraction of (32) with color projection operator $\hat{\mathcal{P}}_{\rm J}^{\rm bb',\,aa'}$ for each projection J we obtain the leading order evolution equation:

$$f_{\rm i,J}(v^2, k^2, k_\perp^2, \hat{x}) = f_{0,\rm i,J}(v^2, k^2, k_\perp^2, \hat{x}) + \frac{\lambda_{\rm J}\alpha_s(k^2)}{\pi}\int\frac{d^4q}{2\pi q^4}\delta((q-k)^2)$$
$$\theta(qn-kn)\theta(vn-qn) \cdot V_{\rm i}(q^2, k^2, z) f_{\rm i,J}(v^2, q^2, q_\perp^2, \frac{\hat{x}}{\hat{z}})\,, \tag{34}$$

where i and J are the spin and the color projection indices correspondingly. In the new variables it looks as follows:

$$f_{\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) = f_{0,\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) + \frac{\lambda_{\mathrm{J}}\alpha_s(k^2)}{\pi}\int\frac{d\hat{z}}{\hat{z}^2(1-\hat{z})^2}\frac{d^2\vec{q}_\perp}{\pi}\frac{dq^2}{q^4}\vec{q}_\perp^{\,\prime 2}$$
$$\theta(1-\hat{z})\theta(\hat{z}-\hat{x}) \cdot \delta\left(\mathcal{A}(q^2,k^2,\hat{z},\vec{q}_\perp^{\,\prime 2})\right)P_{\mathrm{i}}(\hat{z})f_{\mathrm{i,J}}(v^2,q^2,q_\perp^2,\frac{\hat{x}}{\hat{z}})\ , \quad (35)$$

$$P_1(\hat{z}) = \frac{1}{1-\hat{z}}+\frac{1}{\hat{z}}-2+\hat{z}(1-\hat{z})\ , \quad (36)$$

$$P_2(\hat{z}) = \frac{1}{1-\hat{z}}+1-2\hat{z}\ , \quad (37)$$

$$\mathcal{A}(q^2,k^2,\hat{z},\vec{q}_\perp^{\,\prime 2}) = q^2-\frac{k^2}{\hat{z}}-\frac{\vec{q}_\perp^{\,\prime 2}}{\hat{z}(1-\hat{z})}\ , \quad (38)$$

$$\vec{q}_\perp^{\,\prime} = \vec{k}_\perp - \hat{z}\vec{q}_\perp\ ,\ \hat{z} = kn/qn\ ,\ \hat{x}=kn/vn\ . \quad (39)$$

For the radiation of real gluons we have additional conditions $(v-q)^2 \geq 0$ and $(v-k)^2 \geq 0$, which lead to inequalities:

$$a_q \leq -q_\perp^2\frac{\hat{z}}{1-\hat{z}} < 0\ ,\ a_k \leq -k_\perp^2\frac{\hat{x}}{1-\hat{x}} < 0\ , \quad (40)$$

$$q_\perp^2 \leq -q^2(1-\hat{z})\ ,\ k_\perp^2 \leq -k^2(1-\hat{x})\ , \quad (41)$$

$$a_k = k^2+k_\perp^2\ ,\ a_q = q^2+q_\perp^2\ ,\ a_\perp = k_\perp^2+q_\perp^2-2k_\perp q_\perp\cos\phi\ , \quad (42)$$

for $v^2=0$, $v_\perp=0$.

We can rewrite the above equation in different forms. To obtain the first one, which is convenient for numerical solution, we should integrate (35) in q^2 and change $q_\perp$ to $q'_\perp$:

$$f_{\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) = f_{0,\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) + \frac{\lambda_{\mathrm{J}}\alpha_s(k^2)}{\pi}\int\frac{d\hat{z}}{\hat{z}^2}P_{\mathrm{i}}(\hat{z})\int\frac{dq_\perp^{\prime 2}q_\perp^{\prime 2}}{\left(k^2(1-\hat{z})+q_\perp^{\prime 2}\right)^2}$$
$$\int\frac{d\phi'}{\pi}f_{\mathrm{i,J}}(v^2,\frac{k^2}{\hat{z}}+\frac{q_\perp^{\prime 2}}{\hat{z}(1-\hat{z})},\frac{q_\perp^{\prime 2}+k_\perp^2-2q_\perp^\prime k_\perp\cos\phi'}{\hat{z}^2},\frac{\hat{x}}{\hat{z}})\ . \quad (43)$$

Here ϕ' is the angle between $\vec{k}_\perp$ and $\vec{q}'_\perp$. Region of integration is defined by inequalities:

$$\hat{x} \leq \hat{z} \leq 1\ , \quad (44)$$

$$-1 \leq \cos\phi' \leq 1\ , \quad (45)$$

$$q_\perp^{\prime 2} - 2\beta_2 q'_\perp\cos\phi' + \beta_1 \leq 0\ , \quad (46)$$

$$\beta_1 = \frac{k_\perp^2+k^2\hat{z}(1-\hat{z})}{1+\hat{z}}\ ,\ \beta_2 = \frac{k_\perp}{1+\hat{z}}\ . \quad (47)$$

Inequality (46) is obtained from the first one of (41). The above conditions could be resolved. The first region is

$$\hat{x} \leq \hat{z} \leq \sqrt{1-\frac{k_\perp^2}{-k^2}}\ , \quad (48)$$

$$\left(\beta_2-\sqrt{\beta_2^2-\beta_1}\right)^2 \leq q_\perp^{\prime 2} \leq \left(\beta_2+\sqrt{\beta_2^2-\beta_1}\right)^2\ , \quad (49)$$

$$1 \geq \cos\phi' \geq \frac{q_\perp^{\prime 2}+\beta_1}{2\beta_2 q'_\perp}\ , \quad (50)$$

the second one is

$$\beta_1 < 0 \Rightarrow k_\perp^2 \leq \frac{-k^2}{4}\ , \quad (51)$$

$$\max\left[\hat{x}\ ,\ \frac{1}{2}\left(1-\sqrt{1-\frac{4k_\perp^2}{-k^2}}\right)\right] \leq \hat{z} \leq \frac{1}{2}\left(1+\sqrt{1-\frac{4k_\perp^2}{-k^2}}\right)\ , \quad (52)$$

$$0 \leq q_\perp^{\prime 2} \leq \left(\beta_2-\sqrt{\beta_2^2-\beta_1}\right)^2\ , \quad (53)$$

$$1 \geq \cos\phi' \geq -1\ . \quad (54)$$

For the ordering in four-momentum squared and taking into account the initial condition we should also add

$$-q_0^2 \geq q^2 \geq k^2 \Rightarrow (-q_0^2\hat{z} - k^2)(1-\hat{z}) \geq q'^2_\perp \geq -k^2(1-\hat{z})^2 \; . \qquad (55)$$

If we integrate in the azimuthal angle in (35), we will get the alternative form of the evolution equation

$$f_{\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) = f_{0,\mathrm{i,J}}(v^2,k^2,k_\perp^2,\hat{x}) + \frac{\lambda_{\mathrm{J}}\alpha_s(k^2)}{\pi}\int_{\hat{x}}^1 \frac{d\hat{z}}{\hat{z}^3}P_{\mathrm{i}}(\hat{z})$$

$$\int_{k^2}^{-q_0^2}\frac{dq^2}{q^4}(q^2\hat{z}-k^2)\int_{q^2_{\perp-}}^{q^2_{\perp+}}\frac{dq_\perp^2}{\pi\sqrt{(q^2_{\perp+}-q^2_\perp)(q^2_\perp-q^2_{\perp-})}}f_{\mathrm{i,J}}(v^2,q^2,q_\perp^2,\frac{\hat{x}}{\hat{z}}) \; , \qquad (56)$$

$$q^2_{\perp\pm} = \frac{\left(\sqrt{(1-\hat{z})(q^2\hat{z}-k^2)} \pm k_\perp\right)^2}{\hat{z}^2} \; . \qquad (57)$$

It is possible to write the equation for the moments of corresponding functions. Let

$$f^N_{\mathrm{i,J}}(v^2,k^2,k_\perp^2) = \int_0^1 \hat{x}^{N-1} f_{\mathrm{i,J}}(v^2,k^2,k^2_\perp,\hat{x}) \; . \qquad (58)$$

Then after the integration in $\hat{z}$ we have

$$\begin{aligned} f^N_{\mathrm{i,J}}(v^2,k^2,k_\perp^2) &= f^N_{0,\mathrm{i,J}}(v^2,k^2,k_\perp^2) + \frac{\lambda_{\mathrm{J}}\alpha_s(k^2)}{\pi}\int dq_\perp^2 \int \frac{dq^2}{q^4} \\ f^N_{\mathrm{i,J}}(v^2,q^2,q_\perp^2) &\cdot \int_0^\pi \frac{d\phi}{\pi} K^N_{\mathrm{i}}(a_k,a_q,k_\perp^2,q_\perp^2,\cos\phi) \; . \end{aligned} \qquad (59)$$

For the kernel of the equation (59) we have

$$\begin{aligned} K^N_{\mathrm{i}}(a_k,a_q,k_\perp^2,q_\perp^2,\cos\phi) &= \sum_{n=1,2} \hat{z}_n^{*\,N-1}\theta(-q^2(1-\hat{z}^*_n)-q^2_\perp) \cdot \\ &\cdot \quad \frac{(k_\perp^2 + \hat{z}_n^{*\,2}q_\perp^2 - 2k_\perp q_\perp \hat{z}^*_n\cos\phi)P_{\mathrm{i}}(\hat{z}^*_n)}{|\mathcal{A}'(\hat{z}^*_n)|\hat{z}^*_n(1-\hat{z}^*_n)^2} \; , \end{aligned} \qquad (60)$$

$$\hat{z}^*_{1,2} = \frac{a_k + a_q - a_\perp \pm \lambda^{1/2}}{2a_q} \; , \qquad (61)$$

$$\frac{1}{(1-\hat{z}^*_{1,2})^2} = \frac{\left(a_q - a_k + a_\perp \pm \lambda^{1/2}\right)^2}{4a_\perp^2} \; , \qquad (62)$$

$$\mathcal{A}'(\hat{z}^*_{1,2}) \equiv \left.\frac{d\mathcal{A}}{d\hat{z}}\right|_{\hat{z}=\hat{z}^*_{1,2}} = \frac{\pm 2a_q^2\lambda^{1/2}}{a_k^2 - a_\perp(a_q - a_\perp \pm \lambda^{1/2}) - a_k(a_q + 2a_\perp \mp \lambda^{1/2})} \; , \qquad (63)$$

$$\lambda \equiv \lambda(a_k,a_q,a_\perp) = (a_k + a_q - a_\perp)^2 - 4a_k a_q \; , \qquad (64)$$

$$\frac{(k_\perp^2 + \hat{z}^{*\,2}_{1,2}q_\perp^2 - 2k_\perp q_\perp \hat{z}^*_{1,2}\cos\phi)P_{\mathrm{i}}(\hat{z}^*_{1,2})}{|\mathcal{A}'(\hat{z}^*_{1,2})|} = \mathcal{B}^{\mathrm{i}} \pm \lambda^{1/2}\mathcal{V}^{\mathrm{i}} \; , \qquad (65)$$

$$\mathcal{B}^{\mathrm{i}} = \sum_j \mathcal{B}^{\mathrm{i}}{}_j(\cos\phi)^j \; , \; \mathcal{V}^{\mathrm{i}}\lambda = \sum_j \mathcal{V}^{\mathrm{i}}{}_j(\cos\phi)^j \; , \qquad (66)$$

$$\mathcal{B}^{\mathrm{i}}{}_j \equiv \mathcal{B}^{\mathrm{i}}{}_j(k^2,q^2,k_\perp^2,q_\perp^2) \; , \; \mathcal{V}^{\mathrm{i}}{}_j \equiv \mathcal{V}^{\mathrm{i}}{}_j(k^2,q^2,k_\perp^2,q_\perp^2) \; ,$$

Coefficients are the following:

$$\begin{aligned} \mathcal{B}^1{}_0 &= -\frac{q_\perp^2(q_\perp^2-k^2)^5}{2(q^2+q_\perp^2)^6} + \frac{q_\perp^2(q_\perp^2-k^2)^3(k^2+4k_\perp^2+3q_\perp^2)}{2(q^2+q_\perp^2)^5} + \\ &+ \frac{(k^2-q_\perp^2)}{2(q^2+q_\perp^2)^4}\cdot\left((k^2+k_\perp^2)^2(k_\perp^2+2q_\perp^2) - \right. \end{aligned}$$

$$
\begin{aligned}
&- \;\; 2(k^2+k_\perp^2)(k_\perp^2+q_\perp^2)(k_\perp^2+2q_\perp^2)+(k_\perp^2+q_\perp^2)^2(k_\perp^2+5q_\perp^2)\big)+ \\
&+ \;\; \frac{-k_\perp^2(k^2+k_\perp^2)^2-4q_\perp^2(k^2+k_\perp^2)(k_\perp^2+q_\perp^2)+(k_\perp^2+q_\perp^2)^2(k_\perp^2+5q_\perp^2)}{2(q^2+q_\perp^2)^3}+ \\
&+ \;\; \frac{2k^2k_\perp^2+(k^2-4k_\perp^2)q_\perp^2-3q_\perp^4}{2(q^2+q_\perp^2)^2}+\frac{q_\perp^2}{2(q^2+q_\perp^2)}\,, \qquad (67)
\end{aligned}
$$

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\begin{aligned}
\mathcal{B}^1{}_1 \;=\;& \frac{5k_\perp q_\perp^3(q_\perp^2-k^2)^4}{(q^2+q_\perp^2)^6}-\frac{k_\perp q_\perp(q_\perp^2-k^2)^2(k^4+2q_\perp^2(6k_\perp^2-k^2)+13q_\perp^4)}{(q^2+q_\perp^2)^5}+ \\
&+ \;\; \frac{k_\perp q_\perp}{(q^2+q_\perp^2)^4}\big((k^2+k_\perp^2)^3+3(k^2+k_\perp^2)^2(k_\perp^2+2q_\perp^2)- \\
&- \;\; 3(k^2+k_\perp^2)(k_\perp^2+q_\perp^2)(3k_\perp^2+7q_\perp^2)+(k_\perp^2+q_\perp^2)^2(5k_\perp^2+17q_\perp^2)\big)- \\
&- \;\; \frac{k_\perp q_\perp\big(2(k^2+k_\perp^2)^2-4(k^2+k_\perp^2)(k_\perp^2+2q_\perp^2)+(k_\perp^2+q_\perp^2)(5k_\perp^2+13q_\perp^2)\big)}{(q^2+q_\perp^2)^3}+ \\
&+ \;\; \frac{k_\perp q_\perp(4k_\perp^2+5q_\perp^2)}{(q^2+q_\perp^2)^2}-\frac{k_\perp q_\perp}{(q^2+q_\perp^2)}\,, \qquad (68)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{B}^1{}_2 \;=\;& \frac{20k_\perp^2q_\perp^4(k^2-q_\perp^2)^3}{(q^2+q_\perp^2)^6}-\frac{4k_\perp^2q_\perp^2(k^2-q_\perp^2)(2k^4+q_\perp^2(6k_\perp^2-7k^2)+11q_\perp^4)}{(q^2+q_\perp^2)^5}- \\
&- \;\; \frac{6k_\perp^2q_\perp^2(7q_\perp^4-5k^2q_\perp^2+k_\perp^2(5q_\perp^2-3k^2))}{(q^2+q_\perp^2)^4}+\frac{2q_\perp^2k_\perp^2(11q_\perp^2+3k_\perp^2-4k^2)}{(q^2+q_\perp^2)^3}- \\
&- \;\; \frac{4q_\perp^2k_\perp^2}{(q^2+q_\perp^2)^2}\,, \qquad (69) \\
\mathcal{B}^1{}_3 \;=\;& \frac{40k_\perp^3q_\perp^5(q_\perp^2-k^2)^2}{(q^2+q_\perp^2)^6}-\frac{8k_\perp^3q_\perp^3(3k^4+2(k_\perp^2-5k^2)+9q_\perp^4)}{(q^2+q_\perp^2)^5}+ \\
&+ \;\; \frac{4k_\perp^3q_\perp^3(11q_\perp^2+4k_\perp^2-3k^2)}{(q^2+q_\perp^2)^4}-\frac{12k_\perp^3q_\perp^3}{(q^2+q_\perp^2)^3}\,, \qquad (70) \\
\mathcal{B}^1{}_4 \;=\;& \frac{40k_\perp^4q_\perp^6(k^2-q_\perp^2)}{(q^2+q_\perp^2)^6}+\frac{8k_\perp^4q_\perp^4(7q_\perp^2-4k^2)}{(q^2+q_\perp^2)^5}-\frac{16k_\perp^4q_\perp^4}{(q^2+q_\perp^2)^4}\,, \qquad (71) \\
\mathcal{B}^1{}_5 \;=\;& \frac{16k_\perp^5q_\perp^7}{(q^2+q_\perp^2)^6}-\frac{16k_\perp^5q_\perp^5}{(q^2+q_\perp^2)^5}\,, \qquad (72) \\
\mathcal{V}^1{}_0 \;=\;& \frac{q_\perp^2(k^2-q_\perp^2)^6}{(q^2+q_\perp^2)^6}-\frac{q_\perp^2(k^2-q_\perp^2)^4(k^2+3k_\perp^2+2q_\perp^2)}{(q^2+q_\perp^2)^5}+ \\
&+ \;\; \frac{(k^2-q_\perp^2)^2(8q_\perp^6-2k^2q_\perp^4+9q_\perp^2k_\perp^4+3k^4q_\perp^2+k_\perp^2(k^4+2k^2q_\perp^2+15q_\perp^4))}{2(q^2+q_\perp^2)^4}- \\
&- \;\; \frac{(q_\perp^2+k_\perp^2)(k^6+2k^4k_\perp^2+q_\perp^2(k^4-5k^2k_\perp^2+k_\perp^4)+q_\perp^4(5k_\perp^2-6k^2)+5q_\perp^6)}{(q^2+q_\perp^2)^3}+ \\
&+ \;\; \frac{2k_\perp^6+k_\perp^4(2k^2+7q_\perp^2)+3k_\perp^2(k^4-2k^2q_\perp^2+5q_\perp^4)+q_\perp^2(k^4-4k^2q_\perp^2+8q_\perp^4)}{2(q^2+q_\perp^2)^2}- \\
&- \;\; \frac{k_\perp^2(k^2+3q_\perp^2)+2q_\perp^4+2k_\perp^4}{q^2+q_\perp^2}+\frac{1}{2}(2k_\perp^2+q_\perp^2)\,, \qquad (73) \\
\mathcal{V}^1{}_1 \;=\;& \frac{6k_\perp q_\perp^3(k^2-q_\perp^2)^5}{(q^2+q_\perp^2)^6}-\frac{k_\perp q_\perp(k^2-q_\perp^2)^3(k^4+2q_\perp^2(k^2+12k_\perp^2)+21q_\perp^4)}{(q^2+q_\perp^2)^5}+ \\
&+ \;\; \frac{k_\perp q_\perp(k^2-q_\perp^2)}{(q^2+q_\perp^2)^4}\big(2k^6+9k^4k_\perp^2+q_\perp^2(7k^4-24k^2k_\perp^2+18k_\perp^4)+ \\
&+ \;\; q_\perp^4(51k_\perp^2-26k^2)+35q_\perp^6\big)+\frac{k_\perp q_\perp}{(q^2+q_\perp^2)^3}\big(-3k^6+8k^4q_\perp^2-32k^2q_\perp^4+ \\
&+ \;\; 35q_\perp^6+k_\perp^4(21q_\perp^2-13k^2)+k_\perp^2(49q_\perp^4-28k^2q_\perp^2-5k^4)\big)+ \\
&+ \;\; \frac{k_\perp q_\perp(2k^4-3k_\perp^4+6k^2q_\perp^2-21q_\perp^4+2k_\perp^2(5k^2-13q_\perp^2))}{(q^2+q_\perp^2)^2}+
\end{aligned}
$$

$$+ \quad \frac{k_\perp q_\perp(-k^2+4k_\perp^2+7q_\perp^2)}{q^2+q_\perp^2} - k_\perp q_\perp \;, \tag{74}$$

$$\begin{aligned}
\mathcal{V}^1{}_2 \;=\;& \frac{30k_\perp^2 q_\perp^4(q_\perp^2-k^2)^4}{(q^2+q_\perp^2)^6} - \frac{2k_\perp^2 q_\perp^2(q_\perp^2-k^2)^2(5k^4+q_\perp^2(-14k^2+36k_\perp^2)+45q_\perp^4)}{(q^2+q_\perp^2)^5} + \\
+\;& \left(k^6+7k^4(k_\perp^2+q_\perp^2)+q_\perp^2k_\perp^2(3k_\perp^2-22k^2)+q_\perp^4(21k_\perp^2-25k^2)+20q_\perp^6\right)\cdot \\
\cdot\;& \frac{6k_\perp^2 q_\perp^2}{(q^2+q_\perp^2)^4} - \left(6k^4+9k_\perp^4-28k^2q_\perp^2+45q_\perp^4+k_\perp^2(-6k^2+38q_\perp^2)\right)\cdot \\
\cdot\;& \frac{2k_\perp^2 q_\perp^2}{(q^2+q_\perp^2)^3} + \frac{2k_\perp^2 q_\perp^2(11k_\perp^2+18q_\perp^2-2k^2)}{(q^2+q_\perp^2)^2} - \frac{6k_\perp^2 q_\perp^2}{q^2+q_\perp^2} \;,
\end{aligned} \tag{75}$$

$$\begin{aligned}
\mathcal{V}^1{}_3 \;=\;& \frac{80k_\perp^3 q_\perp^5(k^2-q_\perp^2)^3}{(q^2+q_\perp^2)^6} + \frac{8k_\perp^3 q_\perp^3(k^2-q_\perp^2)(5k^4+6q_\perp^2(2k_\perp^2-3k^2)+25q_\perp^4)}{(q^2+q_\perp^2)^5} - \\
-\;& \frac{4k_\perp^3 q_\perp^3(3k^4-39k^2q_\perp^2+50q_\perp^4+k_\perp^2(33q_\perp^2-19k^2))}{(q^2+q_\perp^2)^4} + \\
+\;& \frac{4k_\perp^3 q_\perp^3(25q_\perp^2+9k_\perp^2-8k^2)}{(q^2+q_\perp^2)^3} - \frac{20k_\perp^3 q_\perp^3}{(q^2+q_\perp^2)^2} \;,
\end{aligned} \tag{76}$$

$$\begin{aligned}
\mathcal{V}^1{}_4 \;=\;& \frac{120k_\perp^4 q_\perp^6(k^2-q_\perp^2)^2}{(q^2+q_\perp^2)^6} - \frac{16k_\perp^4 q_\perp^4(5k^4+q_\perp^2(3k_\perp^2-17k^2)+15q_\perp^4)}{(q^2+q_\perp^2)^5} + \\
+\;& \frac{8k_\perp^4 q_\perp^4(20q_\perp^2+6k_\perp^2-7k^2)}{(q^2+q_\perp^2)^4} - \frac{40k_\perp^4 q_\perp^4}{(q^2+q_\perp^2)^3} \;,
\end{aligned} \tag{77}$$

$$\mathcal{V}^1{}_5 \;=\; \frac{96k_\perp^5 q_\perp^7(k^2-q_\perp^2)}{(q^2+q_\perp^2)^6} + \frac{16k_\perp^5 q_\perp^5(9q_\perp^2-5k^2)}{(q^2+q_\perp^2)^5} - \frac{48k_\perp^5 q_\perp^5}{(q^2+q_\perp^2)^4} \;, \tag{78}$$

$$\mathcal{V}^1{}_6 \;=\; \frac{32k_\perp^6 q_\perp^8}{(q^2+q_\perp^2)^6} - \frac{32k_\perp^6 q_\perp^6}{(q^2+q_\perp^2)^5} \;, \tag{79}$$

$$\begin{aligned}
\mathcal{B}^2{}_0 \;=\;& \frac{q_\perp^2(k^2-q_\perp^2)^4}{(q^2+q_\perp^2)^5} - \frac{q_\perp^2((k^2-q_\perp^2)(k^2+6k_\perp^2+5q_\perp^2)}{(q^2+q_\perp^2)^4} + \\
+\;& \frac{5q_\perp^6+4q_\perp^4(2k_\perp^2-k^2)+q_\perp^2(k^4-6k^2k_\perp^2+2k_\perp^4)+2k^4k_\perp^2}{2(q^2+q_\perp^2)^3} + \\
+\;& \frac{-2k_\perp^4+k^2q_\perp^2-3q_\perp^4-k_\perp^2(k^2+3q_\perp^2)}{2(q^2+q_\perp^2)^2} + \frac{k_\perp^2+q_\perp^2}{2(q^2+q_\perp^2)} \;,
\end{aligned} \tag{80}$$

$$\begin{aligned}
\mathcal{B}^2{}_1 \;=\;& \frac{8k_\perp q_\perp^3(k^2-q_\perp^2)^3}{(q^2+q_\perp^2)^5} - \frac{k_\perp q_\perp(k^2-q_\perp^2)(2k^4+q_\perp^2(12k_\perp^2-7k^2)+17q_\perp^4)}{(q^2+q_\perp^2)^4} + \\
+\;& \frac{k_\perp q_\perp(k^4+6k^2q_\perp^2-13q_\perp^4+2k_\perp^2(4k^2-7q_\perp^2))}{(q^2+q_\perp^2)^3} + \frac{k_\perp q_\perp(5q_\perp^2+2k_\perp^2-k^2)}{(q^2+q_\perp^2)^2} - \\
-\;& \frac{k_\perp q_\perp}{q^2+q_\perp^2} \;,
\end{aligned} \tag{81}$$

$$\begin{aligned}
\mathcal{B}^2{}_2 \;=\;& \frac{24k_\perp^2 q_\perp^4(k^2-q_\perp^2)^2}{(q^2+q_\perp^2)^5} - \frac{6k_\perp^2 q_\perp^2(2k^4+q_\perp^2(2k_\perp^2-7k^2)+7q_\perp^4)}{(q^2+q_\perp^2)^4} + \\
+\;& \frac{2k_\perp^2 q_\perp^2(11q_\perp^2+6k_\perp^2-2k^2)}{(q^2+q_\perp^2)^3} - \frac{4k_\perp^2 q_\perp^2}{(q^2+q_\perp^2)^2} \;,
\end{aligned} \tag{82}$$

$$\mathcal{B}^2{}_3 \;=\; \frac{32k_\perp^3 q_\perp^5(k^2-q_\perp^2)}{(q^2+q_\perp^2)^5} + \frac{4k_\perp^3 q_\perp^3(11q_\perp^2-6k^2)}{(q^2+q_\perp^2)^4} - \frac{12k_\perp^3 q_\perp^3}{(q^2+q_\perp^2)^3} \;, \tag{83}$$

$$\mathcal{B}^2{}_4 \;=\; \frac{16k_\perp^4 q_\perp^6}{(q^2+q_\perp^2)^5} - \frac{16k_\perp^4 q_\perp^4}{(q^2+q_\perp^2)^4} \;, \tag{84}$$

$$\mathcal{V}^2{}_0 \;=\; \frac{q_\perp^2(k^2-q_\perp^2)^5}{(q^2+q_\perp^2)^5} + \frac{q_\perp^2(q_\perp^2-k^2)^3(3k^2+10k_\perp^2+7q_\perp^2)}{(q^2+q_\perp^2)^4} +$$

$$
\begin{aligned}
&+ \frac{(k^2-q_\perp^2)(k_\perp^2+q_\perp^2)(k^4+q_\perp^2(5k_\perp^2-k^2)+5q_\perp^4)}{(q^2+q_\perp^2)^3} + \\
&+ \frac{-4k^2q_\perp^4+8q_\perp^6+k_\perp^4(10q_\perp^2-6k^2)+k_\perp^2(15q_\perp^4-4k^2q_\perp^2-3k^4)}{2(q^2+q_\perp^2)^2} + \\
&+ \frac{k_\perp^2(k^2-3q_\perp^2)-2q_\perp^4}{q^2+q_\perp^2} + \frac{1}{2}(k_\perp^2+q_\perp^2) \,, \qquad (85)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}^2{}_1 &= \frac{10k_\perp q_\perp^3(k^2-q_\perp^2)^4}{(q^2+q_\perp^2)^5} - \frac{2k_\perp q_\perp(k^2-q_\perp^2)^2(k^4+q_\perp^2(15k_\perp^2-k^2)+15q_\perp^4)}{(q^2+q_\perp^2)^4} + \\
&+ \frac{k_\perp q_\perp(3k^6+14k^4k_\perp^2+q_\perp^2(3k^4-44k^2k_\perp^2+10k_\perp^4)+q_\perp^4(50k_\perp^2-31k^2)+35q_\perp^6)}{(q^2+q_\perp^2)^3} - \\
&- \frac{k_\perp q_\perp(2k^4+10k_\perp^4-7k^2q_\perp^2+21q_\perp^4+k_\perp^2(k^2+25q_\perp^2))}{(q^2+q_\perp^2)^2} + \frac{k_\perp q_\perp(5k_\perp^2+7q_\perp^2)}{q^2+q_\perp^2} - \\
&- k_\perp q_\perp \,, \qquad (86)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}^2{}_2 &= \frac{40k_\perp^2q_\perp^4(k^2-q_\perp^2)^3}{(q^2+q_\perp^2)^5} - \frac{4k_\perp^2q_\perp^2(k^2-q_\perp^2)(4k^4+q_\perp^2(15k_\perp^2-14k^2)+25q_\perp^4)}{(q^2+q_\perp^2)^4} + \\
&+ \frac{2k_\perp^2q_\perp^2(k^4+26k^2q_\perp^2-45q_\perp^4+k_\perp^2(22k^2-40q_\perp^2))}{(q^2+q_\perp^2)^3} + \\
&+ \frac{2k_\perp^2q_\perp^2(10k_\perp^2+18q_\perp^2-3k^2)}{(q^2+q_\perp^2)^2} - \frac{6k_\perp^2q_\perp^2}{q^2+q_\perp^2} \,, \qquad (87)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}^2{}_3 &= \frac{80k_\perp^3q_\perp^5(k^2-q_\perp^2)^2}{(q^2+q_\perp^2)^5} - \frac{8k_\perp^3q_\perp^3(6k^4+q_\perp^2(5k_\perp^2-21k^2)+20q_\perp^4)}{(q^2+q_\perp^2)^4} + \\
&+ \frac{4k_\perp^3q_\perp^3(10k_\perp^2+25q_\perp^2-7k^2)}{(q^2+q_\perp^2)^3} - \frac{20k_\perp^3q_\perp^3}{(q^2+q_\perp^2)^2} \,, \qquad (88)
\end{aligned}
$$

$$
\mathcal{V}^2{}_4 = \frac{80k_\perp^4q_\perp^6(k^2-q_\perp^2)}{(q^2+q_\perp^2)^5} + \frac{8k_\perp^4q_\perp^4(15q_\perp^2-8k^2)}{(q^2+q_\perp^2)^4} - \frac{40k_\perp^4q_\perp^4}{(q^2+q_\perp^2)^3} \,, \qquad (89)
$$

$$
\mathcal{V}^2{}_5 = \frac{32k_\perp^5q_\perp^7}{(q^2+q_\perp^2)^5} - \frac{32k_\perp^5q_\perp^5}{(q^2+q_\perp^2)^4} \,. \qquad (90)
$$

We can use representation of the equation in the b-space, if we introduce Fourier transform of $f_{i,J}$ and make also transform to the moment space:

$$
\tilde{f}^N_{i,J}(v^2,k^2,b^2) = \int \frac{d^2\vec{k}_\perp}{(2\pi)^2} e^{\imath\vec{b}\vec{k}_\perp} f^N_{i,J}(v^2,k^2,k_\perp^2) \,, \qquad (91)
$$

$$
f^N_{i,J}(v^2,k^2,k_\perp^2) = \int d^2\vec{b}\, e^{-\imath\vec{b}\vec{k}_\perp} \tilde{f}^N_{i,J}(v^2,k^2,b^2). \qquad (92)
$$

In this case equation looks more simple

$$
\begin{aligned}
&\tilde{f}^N_{i,J}(v^2,k^2,b^2) = \tilde{f}^N_{0,i,J}(v^2,k^2,b^2) + \frac{\lambda_J\alpha_s(k^2)}{\pi}\int_0^1 d\hat{z}\ \hat{z}^{N-1}P_i(\hat{z}) \cdot \\
&\cdot \int_{k^2}^{-q_0^2} \frac{dq^2}{q^4}(\hat{z}q^2-k^2)J_0\left(b\sqrt{(1-\hat{z})(\hat{z}q^2-k^2)}\right)\tilde{f}^N_{i,J}(v^2,q^2,\hat{z}^2b^2). \qquad (93)
\end{aligned}
$$

2.3 Semi-inclusive central production

For the semi-inclusive central production (see Fig. 3) we should contract two SU(3) tensors of the type (33) and sum in the spin and color indices. The cross-section is proportional to

$$
\frac{1}{2}\sum_{\mathrm{i}=1,2}\sum_{\mathrm{J}} \mathrm{C_J f_{i,J}}(\mathrm{v}_1^2,\mathrm{k}_1^2,\mathrm{k}_{1\ \perp}^2,\hat{\mathrm{x}}_1)\mathrm{f_{i,J}}(\mathrm{v}_2^2,\mathrm{k}_2^2,\mathrm{k}_{2\ \perp}^2,\hat{\mathrm{x}}_2). \qquad (94)
$$

It is clear from the properties of SU(3) coefficients, that nonsinglet contributions cancels in the above sum.

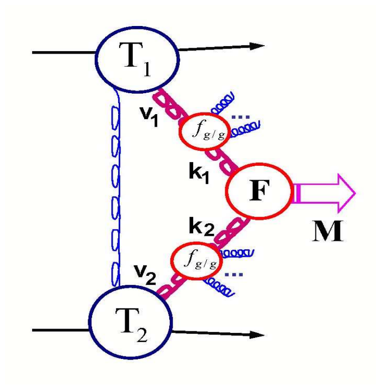

Figure 3: Semi-inclusive process (amplitude without rescattering corrections).

3 Conclusion

In this paper we present the equation for gUPDFs in different forms. The main difference from other approaches is that in our case the main "hard" scale is equal to $-k^2$, and it is not introduced by hand. Solution of this equation could be found numerically. Futher investigations will be presented elsewhere.

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