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FROISSART–MARTIN BOUND IN SPACES WITH COMPACT EXTRA DIMENSIONS

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We generalize the Froissart–Martin upper bound for the scattering amplitude in spaces with compactified extra dimensions.

Geometrically the Froissart–Martin upper bound in the Minkowski space–time of arbitrary dimension[1,2] D

$$\sigma_{\text{tot}}^{D}(s) \leq \text{const}(R_0(s))^{D-2} \tag{1}$$

defines the maximal "transverse area" at which colliding particles effectively feel each other.

The question arises what happens with Eq. (1) if some of $D-1$ spatial[a] dimensions are compactified?

We find more transparent from the physics point of view to use impact parameter representation for the scattering amplitude,

$$T(s, \mathbf{k}_T) \simeq 4s \int d^{D-2} B e^{i\mathbf{k}_T \mathbf{B}} \tilde{T}(s, \mathbf{B})\,, \tag{2}$$

where $\sqrt{s}$ is, as usual, the c.m.s. collision energy, and $k_T^2 \simeq -t$ is related to the transferred momentum.

From the unitarity condition

$$\operatorname{Im} \tilde{T}(s, \mathbf{B}) \geq |\tilde{T}(s, \mathbf{B})|^2\,,$$

analyticity in the t-plane with a nearest singularity at $t = t_0 > 0$, and polynomial boundedness, $|T| < s^N$, one easily recovers the bound (1), with $R_0(s) \simeq \frac{N \log s}{\sqrt{t_0}}$.

Let us compactify for simplicity one transverse dimension in such a way that with $\mathbf{B} \equiv (\mathbf{b}, b_{D-2} \equiv \beta)$ one has

$$\tilde{T}(s, \mathbf{b}, \beta + 2\pi R) = \tilde{T}(s, \mathbf{b}, \beta)\,,$$

[a]We do not consider time-like extra dimensions (moreover compactified), as in this case microcausality, a key ingredient for analyticity, become ambiguous if not worse.

where R is the compactification scale. Conjugated integration in k_{D-2} converts into the sum

$$\sum_{n=-\infty}^{\infty} \frac{e^{in\beta/R}}{2\pi R}.$$

$(D-2)$th component now plays the role of the source of an additional mass spectrum (twice degenerated under $n \to -n$) adding to the usual mass^2 the term n^2/R^2. If R is small (as it seems to be the case in reality), then these modifications to the spectrum do not influence the nearest singularity, and, hence, analyticity.

Below we consider the amplitude for scattering of "light" particles with $n = 0$, which presumably correspond to the usual observed particles. In this case we have from usual arguments

$$\begin{aligned} \operatorname{Im} T(s,0) &\le 4s \int d^{D-3}b \int_{-\pi R}^{\pi R} d\beta \Theta(R_0^2 - b^2 - \beta^2) \\ &= 4s \frac{\Gamma((D-3)/2)}{2\pi^{D-3}/2} \cdot 2\pi R_0^{D-2}(s) \Phi(R_0, R, D)\,, \end{aligned} \tag{3}$$

where

$$\begin{aligned} \Phi = \int_0^1 d\xi\, \xi^{D-4}(1-\xi^2)^{1/2} \Theta\left(\xi - \sqrt{\max[0, (1 - \pi^2 R^2/R_0^2(s))]}\right) \\ + \frac{\pi R}{R_0(s)} \cdot \frac{(1 - \pi^2 R^2/R_0^2(s))_+^{D-3}}{D-3}\,. \end{aligned}$$

At $R \ll R_0(s)$ we get

$$\operatorname{Im} T(s,0) \le \text{const} \cdot s \cdot R_0^{D-3}(s) \cdot R\,. \tag{4}$$

In the opposite, somewhat academic, limit $R/R_0 \to \infty$ we come back to Eq. (1)

$$\operatorname{Im} T(s,0) \le \text{const} \cdot s \cdot R_0^{D-2}(s)\,.$$

It is evident that the above derivation can be readily generalized to more than one compact extra dimension. Furthermore all the inference can be made more rigorous mathematically.

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References

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