Explicit Chiral Symmetry Breaking as a Premise of the Cross-Sections' Rise

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Abstract

We argue that if QCD yields a theory of interacting hadrons then explicit chiral symmetry breaking is a necessary condition for infinitely rising cross-sections. Otherwise cross-sections go to zero at high energies.

On the eve of the LHC operation a special consideration is given to measurements of the total and elastic cross-sections in pp collision at yet unattainable energies up to $10-14\,TeV$ in the c.m.s. All previous measurements since pioneering studies of K^+p interactions at $E_{lab}=30-60\,GeV$ [1] have shown a steady rise both of total and elastic cross-sections till the Tevatron and RHIC energy region. Cosmic rays data (large errors though) also witness in favor of rising cross-sections at least up to several tens TeV. One could ask a somewhat academic question: "If such a rise will go on infinitely?" Certainly, it seems almost evident that at high energy enough when it will not make sense to distinguish among different forces (and now we prescribe the rise in question to the strong interaction) the situation can drastically change. Nonetheless it seems to be of some use to try to understand general features of QCD as the underlying theory for strong interactions of hadrons. In the low-energy sector it proves useful to consider even quarkless theory as a training ground for the study of confinement, estimates of the glueball masses etc (mostly in lattice QCD). Another approximation, QCD with massless quarks, also proves to be very close to reality in the sector of light hadrons. For instance, the nucleon mass changes insignificantly when adding current quark mass terms [2]. So, it is tempting to assume that at high energies these approximations will work at least not worse.

We will try to check such an assumption [3]. If we deal with Quantum Gluodynamics which supposedly results in a theory of colorless massive hadrons - glueballs - interacting with each other, then we are with a single mass scale Λ_{QCD}, a clear imprint of the "dimensional transmutation". All glueball masses (including resonances) are multiples of the fundamental scale

$$M_i^2 = c_i \Lambda_{\mathrm{QCD}}^2,$$

Coefficients c_i are pure numbers. To reveal the provenance of the fundamental scale Λ_{QCD} from the underlying theory we recall that from the renormalization group considerations we have the following relationship with coupling α_s and mass scale μ (renormalization scale)

$$\Lambda_{\mathrm{QCD}}^2 = \mu^2 \exp(-K(\alpha_s)), \qquad (1)$$

$$dK(\alpha_s)/d\alpha_s = 1/\beta(\alpha_s),$$

$$K(\alpha_s) \sim 1/\beta_0\alpha_s + O(\log(1/\alpha_s)) \text{ at } \alpha_s \to 0.$$

Nonanalytic dependence at small α_s is similar to the superconductivity energy gap coupling dependence. In the present context one may argue that parameter Λ_{QCD} is scheme dependent.

However, from the evident property of the physical hadronic amplitudes to be both RG and scheme invariant we are free to use any convenient scheme. In the same way we use a convenient frame for relativistic invariant quantities. We can also refer to paper [4] where a scheme independent expression for the parameter $\Lambda_{\rm QCD}$ has been obtained. Let us look now what happens if we take the free-field limit $\alpha_s \to 0$. As is easily seen from Eq.(1), all masses go to zero. Physically it means that "glueballs" degenerate into a set of systems of collinear gluons which are evidently massless. On the other hand it is also evident that any scattering amplitude vanishes in the free-field limit: no interaction occurs. Take for example an elastic scattering amplitude at $t = 0$. We remind that in the confined gluodynamics no massless hadrons take place (due to the trace anomaly), so that $t = 0$ is an analyticity point. From the dimensional reasons the amplitude is of the form

$$T(s, 0) = \Phi(s/\Lambda^2_{\rm QCD}; \{c_i\}),$$

where $\{c_i\}$ are pure numbers , in general complex, and encode the physical spectrum and depend on spins etc. The rest of the argument is almost trivial. One can see that the free-field limit is identical to the infinite energy limit. Therefore the amplitude vanishes at infinite energies. If we take fixed-angle high energy scattering then it is easy to see that we come to the same conclusion. At fixed and non-zero t (in the physical region) one can see that for the imaginary part of the amplitude the conclusion about decreasing holds because

$$ImT(s, t) \leq ImT(s, 0).$$

As to the real part of the amplitude we can only refer to paper

[4] where it has been proved that (at least for some sequence of s)

$$|T(s,t)| \leq |T(s,0)|$$

if the even signature amplitude dominates forward scattering. However the authors of [4] assumed that the total cross-sections can decrease not faster than $s^{-1/2}$ while, in our case, they decrease faster than s^{-1}.

One can see that inclusion of massless quarks does not change the conclusion about the decrease of the amplitudes. The only difference is that , due to spontaneous chiral symmetry breaking, massless Goldstone bosons appear (pions etc)which can in principle spoil analyticity at $t = 0$. One can show, however, that, say, the imaginary part of the pi-nucleon scattering amplitude remains finite at $t = 0$. So, one can consider pi-nucleon total cross-sections which asymptotically drop to zero as well. We have to note also that non-zero chiral condensate $\langle\bar{\psi}\psi\rangle$ (which can dynamically generate quark masses)does not introduce a new scale, independent of $\Lambda_{\rm QCD}$ and cannot invalidate the argument.

The situation changes radically if chiral symmetry is broken explicitly i.e. the Lagrangian contains terms

$$m\bar{\psi}\psi$$

Now we have got two independent RG-invariant mass-scales, the old one, $\Lambda_{\rm QCD}$ and a new one which can be chosen as

$$M^2 = m^2 \exp(L(\alpha_s))$$

where

$$dL(\alpha_s)/d\alpha_s = \gamma_m(\alpha_s)/\beta(\alpha_s)$$

and $\gamma_m(\alpha_s) = -d(\ln m^2)/d\ln\mu^2$ is the "mass anomalous dimension" defined as in [6]. For simplicity we take one flavor. With two mass scales the forward scattering amplitude is of general form

$$T(s,0) = F(s/\Lambda^2_{\rm QCD}, M^2/\Lambda^2_{\rm QCD}).$$

At infinite energy $T \to F(\infty, M^2/\Lambda^2_{\rm QCD})$, while at $\alpha_s \to 0$, $T \to F(\infty,\infty) = 0$ because $M^2 \sim (\frac{1}{\alpha_s})^{\frac{\gamma_{m_0}}{\beta_0}}$ at $\alpha_s \to 0$. So the (massive) free-field limit is generally different from the high-energy limit and we cannot come to any definite conclusion concerning the latter. Nonetheless, one thing is clear: the infinite rise of cross sections with energy is impossible, in the framework of QCD, without an explicit chiral symmetry violation, i.e. without current quark mass terms in the Lagrangian.

Normally the origin of these current quark mass terms is associated with electroweak symmetry breaking and Yukawa couplings of quarks to the Higgs field. From this angle, the rise of hadronic total cross-sections seems to be strangely induced by the electroweak part of the SM. In other words, the rise of hadronic cross-sections is not a purely strong-interaction feature.

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