

# Gravi-Reggeons and trans-Planckian scattering in models with one extra dimension

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The inelastic scattering of the brane fields induced by  $t$ -channel gravi-Reggeons exchanges in the Randall-Sundrum (RS) model with a small curvature  $\kappa$  is considered, and the imaginary part of the eikonal is analytically calculated. It is demonstrated that the results can be obtained from the corresponding formulae previously derived in the Arkani-Hamed—Dimopoulos—Dvali (ADD) model with one extra dimension of the size  $R_c$  by formal replacement  $R_c \rightarrow (\pi\kappa)^{-1}$ . The inelastic cross section for the scattering of ultrahigh neutrino off the nucleon is numerically estimated for the case  $\kappa \ll \bar{M}_5 \sim 1$  TeV, where  $\bar{M}_5$  is a reduced Planck scale in five warped dimensions.

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## I. INTRODUCTION

In our previous papers [1,2], we calculated the contribution of Kaluza-Klein (KK) gravi-Reggeons into the scattering of four-dimensional standard model (SM) particles in a model with  $d$  compact extra spatial dimensions [the Arkani-Hamed—Dimopoulos—Dvali (ADD) model [3]]. The results were applied to the scattering of cosmic neutrinos off nucleons at super-Planckian neutrino energy  $E_\nu$ .

At  $10^8$  GeV  $< E_\nu < 10^{12}$  GeV, the cross sections related with gravity interactions appeared to be compatible with (larger than) SM cross sections at  $d \leq 3 \div 4$ , depending on  $E_\nu$ . The gravitational part of the cross section induced by the gravi-Reggeon exchange rises rapidly with a decrease of  $d$ . For instance, for  $d = 2$ , it is approximately 2 orders of magnitude larger than the SM contribution to the cross section [1], if the gravity scale is chosen to be  $1 \div 2$  TeV.

Unfortunately, present astrophysical bounds [4] rule out the possibility  $d = 2$  and significantly restrict the parameter space for  $d = 3$ . The case  $d = 1$  is completely excluded since a radius of a single extra dimension,  $R_c$ , exceeds the size of the solar system, if we insist that a fundamental gravity scale in five dimensions,  $\bar{M}_5$ , should be 1 TeV or so. It follows from the relation  $R_c^d \sim \bar{M}_{\text{Pl}}^2 / \bar{M}_D^{2+d}$ , with  $\bar{M}_D$  being a  $D$ -dimensional reduced Planck scale ( $D = 4 + d$ ).

However, the above mentioned astrophysical bounds do not apply to the extra dimensions with a warped metric. In the present paper we consider a model of gravity in a slice of a 5-dimensional anti-de Sitter space (AdS<sub>5</sub>) with a single extra dimension compactified to the orbifold  $S^1/Z_2$  (the Randall-Sundrum (RS) model [5,6]). We consider a special case when a curvature of the metric  $\kappa^1$  is much smaller than the gravity scale  $\bar{M}_5$ .

We demonstrate that in such a limit the expression for inelastic cross section for a collision of the brane particles

in *warped five dimensions* can be obtained from the analogous expression previously derived in *five flat dimensions* by a formal substitution  $R_c \rightarrow 1/(\pi\kappa)$ . Then numerical calculations show that the gravity (gravi-Reggeon) contribution to the scattering of the brane fields should dominate the SM contribution even for rather large  $\bar{M}_5$ .

## II. RS MODEL WITH A SMALL CURVATURE

In the RS model, the warped metric is of the form:

$$ds^2 = e^{-2\kappa|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (1)$$

Here  $y = r\theta$  ( $-\pi \leq \theta \leq \pi$ ),  $r$  being a “radius” of extra dimension, and parameter  $\kappa$  defines the scalar curvature of the space.

We are interested in so-called RS1 model [5] which has two 3-dimensional branes with equal and opposite tensions located at the point  $y = \pi r$  (called the *TeV brane*, or *visible brane*) and at  $y = 0$  (referred to as the *Planck brane*). If  $k > 0$ , then the tension on the TeV brane is negative, whereas the tension on the Planck brane is positive. All SM fields are constrained to the TeV 4D brane, while the gravity propagates in all five dimensions (bulk).

From an effective 4-dimensional action one can derive the relation [5]:

$$\bar{M}_{\text{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} (1 - e^{-2\pi\kappa r}), \quad (2)$$

which means that  $\kappa \sim \bar{M}_5 \sim \bar{M}_{\text{Pl}}$  in this case,  $\bar{M}_5$  being a 5-dimensional reduced Planck scale.

The kinetic energy in the visible brane action is not canonically normalized. After rescaling the fields, the warp factor appears in a mass term:

$$m \rightarrow e^{-\pi\kappa r} m. \quad (3)$$

As a result, the masses of the Kaluza-Klein (KK) graviton excitations are given by

$$m_n = x_n |\kappa| e^{-\pi\kappa r}, \quad n = 1, 2, \dots, \quad (4)$$

where  $x_n$  are zeros of the Bessel function  $J_1(x)$ . Note, here

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<sup>1</sup>The Ricci curvature invariant for this AdS<sub>5</sub> space is given by  $\mathcal{R}^{(5)} = -20\kappa^2$ .

and in what follows we are interested in a case  $\kappa r > 1$ , neglecting terms  $\sim e^{-\pi\kappa r}$  with respect to 1.

The interaction Lagrangian on the brane with a negative tension looks like<sup>2</sup>

$$\mathcal{L} = -\frac{1}{\bar{M}_{\text{Pl}}} T^{\mu\nu} h_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)}. \quad (5)$$

Here  $T^{\mu\nu}$  is the energy-momentum tensor of the matter on the brane,  $h_{\mu\nu}^{(n)}$  is the graviton field with a KK-number  $n$ , and

$$\Lambda_\pi = \bar{M}_{\text{Pl}} e^{-\pi\kappa r} \simeq \left(\frac{\bar{M}_5}{\kappa}\right)^{3/2} \frac{m_n}{x_n} \quad (6)$$

is the physical scale on the TeV brane. It can be chosen as small as 1 TeV for a thick slice of the AdS<sub>5</sub>,

$$r \simeq 12/\kappa \simeq 60l_{\text{Pl}}. \quad (7)$$

We see from (5) that couplings of all massive states are suppressed by  $\Lambda_\pi^{-1}$  only, while the zero mode couples with usual strength defined by the reduced Planck mass  $\bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi}$ .

The relation (7) guarantees that the masses of lowest graviton KK excitations (4) are closed to 1 TeV. Thus, the phenomenology of models with nonfactorizable metric is associated with the resonant KK spectrum in the TeV region [7].

Let us note, that because of the warp factor  $e^{-2\kappa r}$  on the TeV brane, the coordinates  $x^\mu$  are not Galilean. One can, however, introduce the Galilean coordinates  $z^\mu = x^\mu e^{-\pi\kappa r}$  and rewrite both the gravitational field and the energy-momentum tensor in these coordinates (see, for instance, reviews [8]). Then the warp factor is equal to 1 at the negative tension brane, and a correct determination of the masses on this brane can be achieved [9]. By calculating the zero mode sector of the effective theory one thus obtains:

$$\bar{M}_{\text{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} (e^{2\pi\kappa r} - 1). \quad (8)$$

In such a case, we have the following mass spectrum on the negative tension brane:

$$m_n = x_n |\kappa|, \quad n = 1, 2, \dots \quad (9)$$

To get  $m_n \sim 1$  TeV, the parameters of the model are usually taken to be  $\kappa \sim \bar{M}_5 \sim 1$  TeV. Because of the relation

$$x_n = \pi \left(n + \frac{1}{4}\right) + \frac{3}{8} \left[\pi \left(n + \frac{1}{4}\right)\right]^{-1} + \text{O}(n^{-2}), \quad (10)$$

the KK states are equally spaced at large  $n$ .<sup>3</sup> The interac-

<sup>2</sup>We do not consider the radion field here because for the high-energy  $t$ -channel exchanges it is irrelevant.

<sup>3</sup>The first four values of  $x_n$  are 3.83, 7.02, 10.17, and 13.32.

tion Lagrangian is defined by Eq. (5) with the expression of  $\Lambda_\pi$  to be<sup>4</sup>

$$\Lambda_\pi \simeq \frac{\bar{M}_5^{3/2}}{\kappa^{1/2}} = \left(\frac{\bar{M}_5}{\kappa}\right)^{3/2} \frac{m_n}{x_n}. \quad (11)$$

The mass scales of the parameters of the RS model are quite different (Planck scale in the first case and TeV scale in the second case), but a particle phenomenology is similar. Indeed, the substitution  $\kappa \rightarrow \kappa e^{-\pi\kappa r}$ ,  $\bar{M}_5 \rightarrow \bar{M}_5 e^{-\pi\kappa r}$  provides us with the same mass spectrum of the massive gravitons and the same coupling of the KK gravitons to the SM fields.

Nevertheless, the correct statement is that the masses of the KK gravitons, as seen by an observer living on the brane with the negative tension, are defined by Eq. (9) [8]. On the other brane, their values are defined by Eq. (4). Moreover, all bulk fields, not only gravitons, look differently to observers on different branes. The observed masses for brane fields coincide with their Lagrangian values and do not depend on coordinate rescaling, if covariant equations and invariant distances are used [11].

Generally speaking, we have *three* dimensional parameters in the RS model: fundamental gravity scale in 5 dimensions,  $\bar{M}_5$ , the curvature scale,  $\kappa$ , and the size of extra dimension,  $r$ . They obey only *two* conditions. Indeed, to get TeV physics, one fixes  $\bar{M}_5$  to be one or few TeV. Then we can regard Eq. (6) as a relation between free parameters  $\kappa$  and  $r$  at fixed value of  $\bar{M}_5$ .<sup>5</sup>

Thus, there is a possibility to consider a case in which  $\kappa$  is larger than  $r^{-1}$ , but is much smaller than  $\bar{M}_5$ . It was recently demonstrated in Ref. [10], where the warp factor in the line element was chosen to be  $e^{2\kappa|y|}$  instead of  $e^{-2\kappa|y|}$ :

$$ds^2 = e^{2\kappa|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2. \quad (12)$$

The brane located at the point  $y = 0$  has now the negative tension.<sup>6</sup> This brane is regarded as the visible brane, while the Planck brane is located at  $y = \pi r$ . The coordinates  $x^\mu$  are Galilean on the visible brane. It is not surprising that the relations (8) and (9) are reproduced in this scheme.

Following [10], the mass splitting ( $\simeq \pi\kappa$ ) can be chosen to be smaller than the energy resolution of collider experiments. We take  $\pi\kappa = 50$  MeV for phenomenological purposes. Then  $\kappa r \simeq 9.7$ , that corresponds to  $r \simeq 0.61 \text{ MeV}^{-1} \simeq 120 \text{ fm}$ , and the mass of the lightest KK excitation is  $m_1 = 60.5 \text{ MeV}$ . The coupling constant,

<sup>4</sup>The KK gravitons have a universal coupling at both small and large  $\kappa r$  [10]. Remember, we do not consider the intermediate region  $\kappa r \sim 1$ .

<sup>5</sup>Some values of  $\bar{M}_5$  and  $\kappa$  which result in an unnaturally large coupling constant  $\Lambda_\pi$  should be avoided in order not to introduce a new mass scale in the theory.

<sup>6</sup>This choice of the warp factor is equivalent to a replacement  $\kappa \rightarrow -\kappa$  in (1), and the branes are interchanged. Note that Eq. (2) turns into Eq. (8) under such a replacement.

$\Lambda_\pi \simeq (\bar{M}_5/1 \text{ TeV})^{3/2} 141 \text{ TeV}$ , is 2 orders of magnitude larger than in the usually adopted phenomenological scheme [7].

There are restrictions on the parameters of the RS model. In Ref. [7], an upper and lower bound on the ratio  $\kappa/\bar{M}_{\text{Pl}}$  was obtained based on Eqs. (2) and (4) (assuming that  $\kappa \sim \bar{M}_5 \sim \bar{M}_{\text{Pl}}$ ). We will derive analogous bounds on the ratio  $\kappa/\bar{M}_5$ , when the SM fields are on the negative tension brane with the Galilean coordinates, and, consequently, relations (8) and (9) are valid (assuming that  $\kappa \lesssim \bar{M}_5 \sim 1 \text{ TeV}$ ).

We exploit the ideas used in the above mentioned paper [7]. The solution for the metric (1) can be trusted if the 5-dimensional scalar curvature,  $\mathcal{R}^{(5)} = -20\kappa^2 e^{2\pi\kappa r}$ , obeys the inequality  $|\mathcal{R}^{(5)}| < \bar{M}_5^2 e^{2\pi\kappa r}$ , that results in the condition  $\kappa/\bar{M}_5 \lesssim 0.2$ .

The D3-brane tension  $\tau$  in the heterotic string theory is given by [12]

$$\tau_3 = \frac{M_s^4}{g_s(2\pi)^3}, \quad (13)$$

where  $M_s = (\alpha')^{-1}$  is the string scale, and  $g_s$  is the string coupling constant. The low-energy action in the strongly coupled heterotic string theory in ten dimensions looks like [12]:

$$S = \int d^{10}x \left[ \frac{M_s^8}{(2\pi)^7 g_s^2} \mathcal{R} + \frac{1}{4} \frac{M_s^6}{(2\pi)^7 g_s} F^2 + \dots \right]. \quad (14)$$

After compactification of ten-dimensional action to four dimensions the coefficient of  $\mathcal{R}$  and  $(1/4)F^2$  should be identified with  $1/(16\pi G_N)$  and  $1/g_G^2$ , respectively, where  $g_G$  is a 4-dimensional gauge coupling taken at the string scale  $M_s$ .<sup>7</sup> Let us first assume that all six extra dimensions are compact ones. By performing T-duality to six dimensions, one then obtains [3]:

$$g_s = \frac{g_G^2}{2\pi}. \quad (15)$$

By compactifying the action (14) to five warp dimensions, we get:

$$\bar{M}_5^3 = \frac{2V_5 M_s^8}{g_s(2\pi)^7}, \quad (16)$$

where  $V_5$  is a volume of five-dimensional manifold with nonfactorizable metric. Let us now assume that the ratio (15) remains valid. Then, taking five extra dimensions to have a common radius  $R_c = M_s^{-1}$ , we find:

$$M_s = \left( \frac{g_G^4}{2} \right)^{1/3} \bar{M}_5. \quad (17)$$

<sup>7</sup>For a possibility to lower the string scale to the TeV region, see Refs. [13].

On the other hand, the tension of the 3-branes in the RS model is [5]

$$|\tau| = 24\bar{M}_3^3 \kappa. \quad (18)$$

Requiring  $|\tau| = \tau_3$ , one gets from (13), (17), and (18) that  $\kappa/\bar{M}_5 \simeq 6.1 \cdot 10^{-4}$  ( $1.3 \cdot 10^{-5}$ ) for  $\alpha_G = g_G^2/4\pi \simeq 0.1$  (0.01). As one can see, the value of the ratio  $\kappa/\bar{M}_5$  depends on which of the SM gauge couplings are chosen to represent  $g_G$ . We take the following region for a phenomenological analysis:

$$10^{-5} \leq \frac{\kappa}{\bar{M}_5} \leq 0.1. \quad (19)$$

For  $\bar{M}_5 = 1 \text{ TeV}$ , Eq. (19) corresponds to  $10 \text{ MeV} \leq \kappa \leq 0.1 \text{ TeV}$ . Remember that the fundamental mass scale is related with the Planck mass by Eq. (8), while the masses of the KK excitations are given by Eq. (9). In what follows, we will be interested in a case when the ratio  $\kappa/\bar{M}_5$  is closed to the lower end of the range (19), and  $\kappa \ll \bar{M}_5 \sim 1 \text{ TeV}$ .

Note, the RS model with the small curvature may be regarded as a small distortion of the compactified flat space with one large extra dimension. Such space warping gives a model which has the ultraviolet properties of the ADD model with a single extra dimensions [10], but it evades the contradiction with available astrophysical bounds.

### III. EIKONAL IN FLAT AND WARP FIVE DIMENSIONS

Now let us consider a scattering of two *pointlike* brane particles (say, lepton-quark or quark-quark scattering) in the trans-Planckian kinematical region

$$\sqrt{s} \gg M_D, \quad s \gg -t, \quad (20)$$

$t = -\mathbf{q}_\perp^2$  being the four-dimensional momentum transfer. More realistic case of neutrino-proton interactions will be studied in the next section.

Regime (20) dominates the whole value of the total, elastic and inelastic cross sections, while the fixed-angle regime ( $s \sim -t$ ) is but a tiny fraction of them.

Unitarity is satisfied automatically if one uses the (exact) eikonal representation for the scattering amplitude:

$$A(s, t) = 2is \int d\mathbf{b} e^{i\mathbf{q}_\perp \cdot \mathbf{b}} [1 - e^{i\chi(s, \mathbf{b})}], \quad (21)$$

where unitarity leads to the condition:

$$\text{Im } \chi(s, \mathbf{b}) > 0. \quad (22)$$

Here  $\mathbf{b}$  means the impact parameter (or a coordinate in the transverse plane to the beam direction).

The approximation we use is so-called Regge-eikonal approximation in which the exchange potential is relativistically generalized to the Reggeized exchanges [14].

In the eikonal approximation an elastic scattering amplitude in the kinematical region (20) is given by the sum

of Reggeized gravitons in  $t$ -channel. So, we assume that both massless graviton and its KK massive excitations lie on linear Regge trajectories. Because of a presence of the extra dimension, we come to splitting of the Regge trajectory (14) into a leading vacuum trajectory

$$\alpha_0(t) \equiv \alpha_{\text{grav}}(t) = 2 + \alpha'_g t, \quad (23)$$

and an infinite sequence of secondary, ‘‘KK-charged,’’ gravi-Reggeons [1,15]:

$$\alpha_n(t) = 2 + \alpha'_g t - \alpha'_g m_n^2, \quad n \geq 1. \quad (24)$$

The string theory implies that the slope of the gravi-Reggeon trajectory is universal for all  $s$ , and  $\alpha'_g = \alpha' = 1/M_s^2$ .

Let us first consider the scattering of the brane fields in a model with  $d$  flat compact extra dimensions [3]. In the ADD model the masses of the KK gravitons are given by  $m_n^2 = n^2/R_c^2$ , where  $n^2 = n_1^2 + \dots + n_d^2$ , and  $R_c$  is the compactification radius of the extra dimensions. The coupling of both zero and massive modes to colliding particles are suppressed by the Planck scale. Therefore, the Born amplitude looks like

$$A_{\text{ADD}}^B(s, t) = \frac{\pi \alpha'_g s^2}{2\bar{M}_{\text{Pl}}^2} \sum_{n_1, \dots, n_d} \left[ i - \cot \frac{\pi}{2} \alpha_n(t) \right] \left( \frac{s}{s_0} \right)^{\alpha_n(t)-2}. \quad (25)$$

As it was shown in Ref. [10], it is ladder diagrams that make a leading contribution of the KK gravitons to the amplitude and result in the eikonal representation for the amplitude, with the eikonal given by

$$\chi(s, b) = \frac{1}{4\pi s} \int dq_{\perp} q_{\perp} J_0(q_{\perp} b) A^B(s, -q_{\perp}^2), \quad (26)$$

where  $b = |\mathbf{b}|$ ,  $q_{\perp} = \mathbf{q}_{\perp}$ , and  $J_0(q_{\perp} b)$  is the Bessel function.

Let us consider the imaginary part of the eikonal in which the zero mode contribution is negligible. The analytical expression for  $\text{Im}\chi(s, b)$  was derived in [1]:

$$\begin{aligned} \text{Im}\chi_{\text{ADD}}(s, b) &= \frac{s\alpha'_g}{16\bar{M}_{\text{Pl}}^2 R_g^2(s)} \exp[-b^2/4R_g^2(s)] \\ &\times \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp[-n^2 \alpha'_g \ln(s/s_0)/R_c^2] \right\}^d, \end{aligned} \quad (27)$$

where

$$R_g(s) = \sqrt{\alpha'_g [\ln(s/s_0) + b_0]} \quad (28)$$

is a gravitational slope. Since  $b_0 = O(1)$ , it can be neglected at large  $s$  [1].

The sum in Eq. (27) is one of the Jacobi  $\theta$ -functions [16]:

$$\theta_3(0, p) = 1 + 2 \sum_{k=1}^{\infty} p^{k^2}, \quad (29)$$

with

$$p = \exp[-\alpha'_g \ln(s/s_0)/R_c^2]. \quad (30)$$

By using unimodular transformation of the  $\theta_3$ -function [16] (known also as Jacobi imaginary transformation) one can obtain the following asymptotic of  $\theta_3(0, p)$  for large extra dimensions:

$$\theta_3(0, p)|_{R_c^2 \gg \alpha'_g \ln(s/s_0)} \simeq \sqrt{\frac{\pi R_c^2}{\alpha'_g \ln(s/s_0)}}. \quad (31)$$

As a result, we get that in a flat metric with  $d$  extra compact dimensions the imaginary part of the eikonal is of the form [1]:

$$\begin{aligned} \text{Im}\chi_{\text{ADD}}(s, b) &\simeq \frac{1}{16\pi^{d/2} [\ln(s/s_0)]^{(2+d)/2}} \frac{s}{\bar{M}_D^2} \left( \frac{M_s}{2M_D} \right)^d \\ &\times \exp[-b^2/4R_g^2(s)]. \end{aligned} \quad (32)$$

Now let us return to the *nonfactorizable* metric (1). According to (5), the Born amplitude is of the form

$$\begin{aligned} A_{\text{RS}}^B(s, t) &= \frac{\pi \alpha'_g s^2}{2\bar{M}_{\text{Pl}}^2} \left[ i - \cot \frac{\pi}{2} \alpha_0(t) \right] \left( \frac{s}{s_0} \right)^{\alpha_0(t)-2} \\ &+ \frac{\pi \alpha'_g s^2}{2\Lambda_{\pi}^2} \sum_{n \neq 0} \left[ i - \cot \frac{\pi}{2} \alpha_n(t) \right] \left( \frac{s}{s_0} \right)^{\alpha_n(t)-2}. \end{aligned} \quad (33)$$

The index  $n$  runs over all negative and positive integers.

Zero mode contribution to the imaginary part of the eikonal [the first term in Eq. (33)] is negligible and can be omitted. Then the total contribution of the massive KK excitations follows from (33):

$$\begin{aligned} \text{Im}\chi_{\text{RS}}(s, b) &= \frac{s\alpha'_g}{16\Lambda_{\pi}^2 R_g^2(s)} \exp[-b^2/4R_g^2(s)] \\ &\times \sum_{n \neq 0} \exp[-\alpha'_g m_n^2 \ln(s/s_0)]. \end{aligned} \quad (34)$$

Let us remember that we are interested in small  $\kappa \ll 1$  TeV. In such a case, the sum in (34) is defined mainly by large  $n$ ,<sup>8</sup> and one can put  $m_n = (n + 1/2)\pi\kappa$  [see Eq. (10)]. Then we can write

<sup>8</sup>The sum in Eq. (34) is effectively cut off, and  $n \leq n_{\text{max}} = (M_s/\pi\kappa)(\ln(s/s_0))^{-1/2} \simeq 2 \cdot 10^4 (\ln(s/s_0))^{-1/2}$  in our case.

$$\begin{aligned}
\sum_{n \neq 0} \exp[-\alpha'_g m_n^2 \ln(s/s_0)] &\simeq \sum_{n=-\infty}^{\infty} \exp[-(n+1/2)^2 \\
&\quad \times (\pi\kappa)^2 \alpha'_g \ln(s/s_0)] \\
&= \sum_{n=-\infty}^{\infty} q^{(n+1/2)^2} \equiv \theta_2(0, q) \\
&= \left(-\frac{\ln q}{\pi}\right)^{1/2} \theta_4(0, e^{\pi^2/\ln q}).
\end{aligned} \tag{35}$$

Here  $\theta_2(0, q)$  and

$$\theta_4(0, v) = 1 + 2 \sum_{k=1}^{\infty} (-1)^k v^{k^2} \tag{36}$$

are the Jacobi  $\theta$ -functions,<sup>9</sup> and variable

$$q = \exp[-(\pi\kappa)^2 \alpha'_g \ln(s/s_0)] \tag{37}$$

is introduced in (35).

As a result, we obtain the following analytical expression for the imaginary part of the eikonal:

$$\begin{aligned}
\text{Im } \chi_{\text{RS}}(s, b) &= \frac{s\alpha'_g}{16\Lambda_\pi^2 R_g^2(s)} \exp[-b^2/4R_g^2(s)] \\
&\quad \times [\pi\kappa^2 R_g^2(s)]^{-1/2} \left[ 1 + 2 \sum_{n=1}^{\infty} (-1)^n \right. \\
&\quad \left. \times \exp(-n^2/\kappa^2 R_g^2(s)) \right].
\end{aligned} \tag{38}$$

If  $\kappa R_g^2(s) \ll 1$ ,<sup>10</sup> then all terms in the sum are exponentially suppressed with respect to unity. As for the leading term in Eq. (38), it can be obtained from the expression for the imaginary part of the eikonal in the ADD model with a single extra dimension by using the following replacements in the KK sector:

$$\bar{M}_{\text{Pl}} \rightarrow \Lambda_\pi, \quad R_c \rightarrow \frac{1}{\pi\kappa}. \tag{39}$$

Indeed, for  $\kappa \ll M_s$ , we get from (38):

$$\begin{aligned}
\text{Im } \chi_{\text{RS}}(s, b)|_{\kappa \ll M_s} &\simeq \frac{1}{16\pi^{1/2} [\ln(s/s_0)]^{3/2}} \frac{sM_s}{\bar{M}_5^3} \\
&\quad \times \exp[-b^2/4R_g^2(s)],
\end{aligned} \tag{40}$$

and  $\text{Im } \chi_{\text{RS}}(s, b)$  (40) coincides with  $\text{Im } \chi_{\text{ADD}}(s, b)$  (32) for  $d=1$  up to a numerical factor  $1/2$ , if we identify 5-dimensional (reduced) Planck scales  $\bar{M}_5$  in both schemes.

Note that the asymptotic of the eikonal (40) does not depend on  $\kappa$  in the limit  $\kappa \ll M_s$ , up to insignificant corrections  $\text{O}(\exp[-M_s^2/\kappa^2 \ln(s/s_0)])$ . This allows us to

<sup>9</sup>In obtaining last term in (35), a unimodular transformation of the  $\theta_2$ -function was used.

<sup>10</sup>Since  $\ln(s/s_0)$  rises slowly in  $s$ , this condition is satisfied if  $\kappa \ll (\alpha'_g)^{-1} = M_s$ .

study the dependence of the gravity induced cross sections on the parameters  $\bar{M}_5$  and  $M_s$ . In what follows, we will use  $s_0 = \alpha'_g$ , a scale motivated by the string theory.

#### IV. ULTRA-HIGH-ENERGY NEUTRINO-NUCLEON SCATTERING INDUCED BY GRAVITATIONAL INTERACTIONS

Let us now apply our results to the scattering of ultra-high-energy cosmic neutrinos off atmospheric nucleons (protons, for a certainty). In the eikonal approximation, the neutrino-proton inelastic cross section is

$$\sigma_{\text{in}}^{\nu p}(s) = \int d^2b \{1 - \exp[-2\text{Im} \chi_{\nu p}(s, b)]\}. \tag{41}$$

with the eikonal defined by

$$\chi_{\nu p}(s, b) = \frac{1}{4\pi s} \int_0^\infty q_\perp dq_\perp J_0(q_\perp b) A_{\nu p}^B(s, -q_\perp^2). \tag{42}$$

In the RS model with the small curvature ( $\kappa \ll \bar{M}_5, M_s$ ), the Born amplitude can be easily calculated by using formulae obtained in Sec. 10:

$$\begin{aligned}
A_{\nu p}^B(s, t) &= \frac{s^2}{2\sqrt{\pi} \bar{M}_5^3 M_s^2} \sum_i \int_{s_0/s}^1 dx x^2 \frac{1}{R_g(sx)} \\
&\quad \times \exp[tR_g^2(sx)] F_i(x, t, \mu^2),
\end{aligned} \tag{43}$$

where  $F_i(x, t, \mu^2)$  is a skewed ( $t$ -dependent) distribution of parton  $i$  ( $i = q, \bar{q}, g$ ) inside the proton. The mass scale in  $F_i(x, t, \mu^2)$  is defined by a large scale induced by gravitational forces,  $\mu = 1/[2R_g(s)]$ . The gravitational interaction radius,  $R_g(s)$ , is introduced above (28).

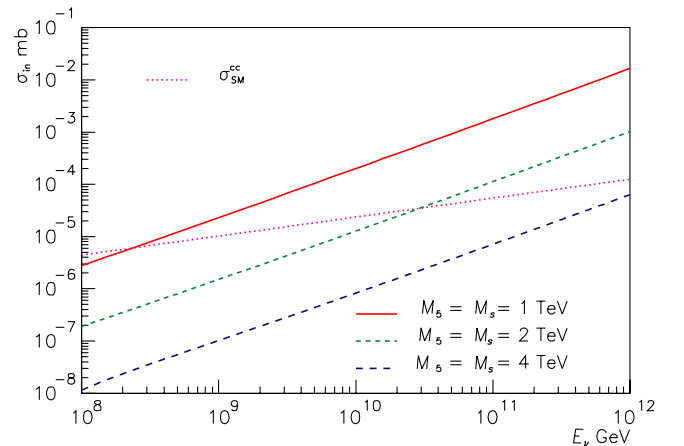


FIG. 1 (color online). The gravitational inelastic neutrino-proton cross-section as a function of the neutrino energy  $E_\nu$  for three different values of the (reduced) fundamental scale  $\bar{M}_5$  and string scale  $M_s$ , which are assumed to be the same. For comparison, the SM charged current neutrino-proton cross section is presented (dotted curve).

Assuming the Regge behavior, we can write

$$F_i(x, t, \mu^2) = f_i(x, \mu^2) \exp[t(r^2 + \alpha'_p \ln(1/x))], \quad (44)$$

where  $\alpha'_p$  is the Pomeron slope, and  $f_i(x, \mu^2)$  is a standard parton distribution function (PDF) of parton  $i$  in momentum fraction  $x$ . We use a set of PDF's from paper [17] based on an analysis of existing deep inelastic data in the next-to-leading order QCD approximation in the fixed-flavor-number scheme. The PDF's are available in the region  $10^{-7} < x < 1$ ,  $2.5 \text{ GeV}^2 < Q^2 < 5.6 \cdot 10^7 \text{ GeV}^2$  [17].

We will use a fit from Ref. [18] for the radius  $r$  and slope of the *hard* Pomeron (remember that  $\mu \sim M_s \sim 1 \text{ TeV}$ ):

$$r^2 = 0.62 \text{ GeV}^{-2}, \quad \alpha'_p = 0.094 \text{ GeV}^{-2}. \quad (45)$$

Since  $r^2 \gg R_g^2(s)$  (at any conceivable  $s$ ), a falloff of the eikonal in impact parameter  $b$  will be mainly defined by strong interactions (namely, by the typical hadronic scale  $r$  of order  $1 \text{ GeV}^{-1}$ ), and not by short-range gravitational forces due to KK gravi-Reggeons.

The inelastic cross-sections induced by gravi-Reggeons are presented in Figs. 1 and 2 for different parameter sets  $(\bar{M}_5, M_s)$ . In both figures, the SM neutrino-proton charged current cross section is also presented. An approximation for the SM cross section valid in the range  $10^7 \text{ GeV} \lesssim E_\nu \lesssim 10^{12} \text{ GeV}$  is taken from Ref. [19].

It is interesting to compare the gravitational inelastic cross section with the black hole production cross section. Let  $\hat{\sigma}$  be the cross section of the black hole production in the neutrino-quark (or neutrino-gluon) subprocess. Then the black hole production cross section in the neutrino-proton collision can be presented in the form

$$\sigma_{\text{bh}}(s) = \int_{(M_{\text{bh}}^{\text{min}})^2/s}^1 dx \hat{\sigma}(\sqrt{xs}) \sum_i f_i(x, \tilde{\mu}^2), \quad (46)$$

where  $s = 2M_p E_\nu$  is an invariant collision energy, with  $\sqrt{xs}$  being a black hole mass  $M_{\text{bh}}$ . The quantity  $M_{\text{bh}}^{\text{min}}$  in

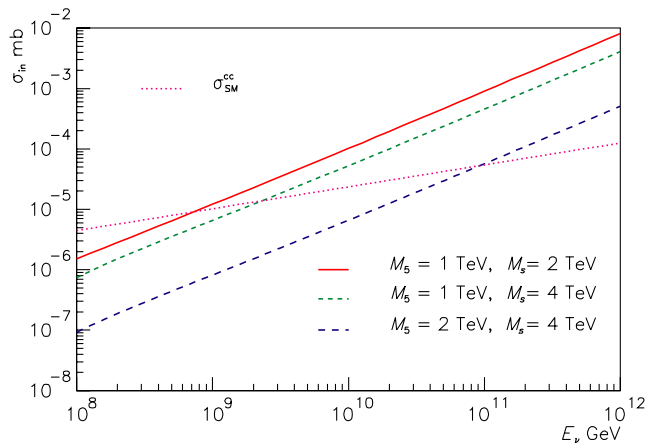


FIG. 2 (color online). The same as in Fig. 1, but the (reduced) fundamental scale  $\bar{M}_5$  is chosen to be less than the string scale  $M_s$ .

(46) is a minimal value of  $M_{\text{bh}}$ . A mass scale in PDF's is chosen to be  $\tilde{\mu} = 1/R_S(M_{\text{bh}})$ .

The cross section  $\hat{\sigma}$  in Eq. (46) is usually taken in a simple geometrical form [20],

$$\hat{\sigma}(E) = \pi R_S^2(E), \quad (47)$$

where  $R_S(E)$  is the size of 5-dimensional Schwarzschild radius [21]:

$$R_S(E) = \sqrt{\frac{2}{3\pi} \frac{E}{M_5^3}}. \quad (48)$$

As in the case of five flat dimensions, we define the fundamental Planck scale  $M_5$  to be related with the *reduced* Planck scale  $\bar{M}_5$  by equation  $M_5 = (2\pi)^{1/3} \bar{M}_5 \approx 1.8 \bar{M}_5$ .

The use of flat space formulae for the black hole production implies that the Schwarzschild radius (48) is much less than the AdS<sub>5</sub> curvature as viewed on the visible brane,  $R_S \ll \kappa^{-1}$ . In its turn, this inequality means

$$M_{\text{bh}} \ll 3\pi^2 x_1 \frac{\Lambda_\pi^2}{m_1}, \quad (49)$$

where  $m_1$  is the mass of the lightest KK gravitons, and  $x_1$  is the first zero of the Bessel function  $J_1(x)$ . Let us stress, the inequality for  $M_{\text{bh}}$  in this form (49) is valid in both scheme (2) and scheme (8). Since in our case with  $\kappa \ll \bar{M}_5 \sim 1 \text{ TeV}$  the value of  $\Lambda_\pi$  is of the order of 100 TeV, while the lightest mass is about 60 MeV [see estimates after Eq. (12)], inequality (49) admits much higher  $M_{\text{bh}}$  than one can have in a usually adopted scheme with  $\kappa \sim \bar{M}_5 \sim \bar{M}_{\text{Pl}}$ , in which  $m_1 \sim \Lambda_\pi \sim 1 \text{ TeV}$ .

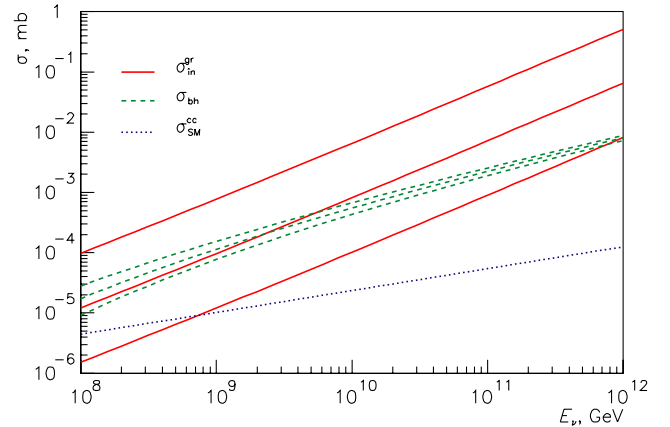


FIG. 3 (color online). The gravitational inelastic neutrino-proton cross-sections (solid lines) vs black hole production cross-sections (dashed lines). The solid curves correspond to  $M_s = 2 \text{ TeV}$  and  $\bar{M}_5 = 0.25 \text{ TeV}, 0.5 \text{ TeV}, 1 \text{ TeV}$  (from the top). The dashed lines correspond to  $M_s = 0.9 \text{ TeV}$  (i.e.  $\bar{M}_5 = 0.5 \text{ TeV}$ ) and  $M_{\text{bh}}^{\text{min}} = 0.5 \text{ TeV}, 1 \text{ TeV}, 2 \text{ TeV}$  (from the top). The SM cross section is also shown (dotted line).

In Fig. 3 we present the black hole production cross section in comparison with the gravitational cross section. As one can see, at  $\bar{M}_5 = 0.5$  TeV, gravi-Reggeon interactions (middle solid line in Fig. 3) can dominate black hole production mechanism at  $E_\nu \gtrsim 4 \cdot 10^9$  GeV (dashed lines).

## V. CONCLUSIONS AND DISCUSSIONS

In order to get a correct interpretation of the KK graviton masses in the RS-like model, one can use the nonfactorizable metric which has an exponentially decreasing warp factor  $\exp(-2\kappa|y|)$  in its 4-dimensional part and then turn to the Galilean coordinates. The SM fields are assumed to be placed on the TeV (visible) brane located at  $y = \pi r$ , where  $r$  is the size of the 5-th dimension. Remember that  $\kappa$  is a measure of the negative constant curvature of the AdS<sub>5</sub> space.

Another way is to choose the exponentially growing warp factor, namely  $\exp(2\kappa|y|)$ , but to place the visible brane at the point  $y = 0$  [10]. In such a case, the coordinates are Galilean from the very beginning. This choice of the warp factor is equivalent to a formal replacement  $\kappa \rightarrow -\kappa$ . Both ways lead to the hierarchy relation  $\bar{M}_{\text{Pl}}^2 \simeq (\bar{M}_5^2/\kappa) \exp(2\pi\kappa r)$ . Thus, one can get a TeV-phenomenology even if  $\kappa \ll \bar{M}_5$ , due to the presence of the large factor  $\exp(2\pi\kappa r)$ , provided  $\bar{M}_5 \sim 1$  TeV and  $\kappa r \approx 10$ .

In the present paper, we have considered the case  $\kappa \ll \bar{M}_5 \sim 1$  TeV and have studied the inelastic scattering of the brane fields induced by gravitational interactions in the  $t$ -channel. Namely, we have summed an infinite set of trajectories (gravi-Reggeons) corresponding to the massive KK gravitons which lie on these trajectories. The imaginary part of the eikonal,  $\text{Im}\chi(s, b)$ , has been analytically calculated. It coincides with the imaginary part of the eikonal derived in the scheme with one flat extra dimension

of the size  $R_c$  [1], after a replacement  $R_c \rightarrow (\pi\kappa)^{-1}$ . It is interesting to note that  $\text{Im}\chi(s, b)$  depends on the 5-dimensional Planck scale  $\bar{M}_5$  and the slope of the gravi-Reggeons  $\alpha'_g$ , but it does not depend on  $\kappa$  in the limit  $\kappa \ll \bar{M}_5$  (up to negligible corrections).

Thus, the scattering of the SM particles in the AdS<sub>5</sub> space with a small curvature looks similar to their scattering in the 5-dimensional flat space. It does not mean, however, that the RS model with small curvature is equivalent to the ADD model with *one* large extra dimension of the size  $R_c^{-1} = (\pi\kappa)$ . Indeed, in the ADD model with the fundamental scale of order of 1 TeV,  $R_c^{-1} \sim 10^{-30/d+6}$ , where  $d$  is the number of compact dimensions. According to this relation, compactification radius  $R_c^{-1} = (\pi\kappa) \approx 50$  MeV can be realized only for  $d = 7$ .

The results have been applied to the calculation of the gravity contribution to the scattering of ultra-high-energy neutrino off the nucleon as a function of the neutrino energy  $E_\nu$ . In particular, we have found that for  $M_5 \approx 1$  TeV (which is equivalent to  $\bar{M}_5 \approx 0.5$  TeV) the gravitational part of the inelastic cross sections appeared to be comparable with (or larger than) the black hole production cross section for  $M_{\text{bh}}^{\text{min}} = 1 \div 2$  TeV in the region  $E_\nu \gtrsim 4 \cdot 10^9$  GeV.

Note, in the model with the flat metric, the gravi-Reggeon cross section for the neutrino-nucleon scattering grows significantly for small  $d$  [1]. Unfortunately, small values of the number of the flat dimensions ( $d \leq 3$ ) are ruled out by the astrophysical bounds [4]. On the contrary, the scheme with the warped metric and one extra dimension is free of these bounds, and rather large cross sections (up to 0.01 – 0.1 mb, at  $E_\nu = 10^{12}$  GeV) are expected in this case. The neutrino-nucleon cross sections will be probed by the Pierre Auger Observatory at the level of SM predictions, taking into account the high statistics to be collected by this experiment in six years of operations [22].

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