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Does the efficiency of high energy collisions depend on a hard scale?

A.V. Kisselev[a], V.A. Petrov[b]

Institute for High Energy Physics, 142281 Protvino, Russia

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Abstract. The multiplicity of charged hadrons in the current fragmentation region of both the c.m.s. and the Breit frame of deep inelastic scattering is calculated and compared with the HERA data. The results are in agreement with Yang's hypothesis that the efficiency of high energy processes increases at larger momentum transfer, although the effect is rather weak numerically at present values of Q^2.

1 Introduction

It seems quite natural to expect that the harder a high energy collision is, the higher is the number of fragments. One of the most suitable and widely explored processes where this phenomenon can be seen is deeply inelastic scattering (DIS) where we have a possibility to change the hard scale (Q^2, gauge boson virtuality) and to detect its influence (if any) on the "efficiency" of the hadronic invariant mass (W). The simplest measure of this efficiency is the multiplicity of secondaries.

In 1969 Yang and his collaborators [1], basing themselves on the "fragmentation picture" of violent collisions, made a qualitative prediction: "... for larger values of the momentum transfer t, the breakup process favors larger multiplicities of hadrons" (at fixed hadronic mass). Early searches for this effect were inconclusive in both theory and experiment [2].

In the framework of QCD a quantitative result has been obtained in [3,4]: it appears that QCD gluon bremsstrahlung leads to an increase of the hadron multiplicities in DIS, with an increase of Q^2 at fixed hadronic mass W, but that this increase is very slow. The distinctive feature of this result is that $\langle n\rangle^{\mathrm{DIS}}(W,Q^2)$ has a finite limit at $Q^2 \to \infty$ and W fixed.

Later, another result has been claimed in [5], which predicted unlimited and quite rapid growth of the hadronic multiplicity $\langle n\rangle^{\mathrm{DIS}}(W,Q^2)$ with Q^2. In the course of the inference of this result it was supposed that the influence of the (non-perturbative) composite structure of the nucleon is negligible while in [3,4] it plays a key role in the slowness of the Q^2 dependence of the multiplicity at fixed W. There is an even more trivial objection. On general grounds, unlimited growth with Q^2 at fixed W is impossible because of the a priori kinematical bound

$$\langle n\rangle^{\mathrm{DIS}}(W,Q^2) \le \frac{W}{m}, \tag{1}$$

where m is some effective mass.

Quite recently, a weak dependence on Q^2 in the framework of the dual parton model was mentioned in [6].

Experimentally a statistically significant effect of the slow growth of $\langle n\rangle^{\mathrm{DIS}}(W,Q^2)$ was established in $\nu(\bar{\nu})p$ interactions [7] and in μ^+p interactions [8]. The results of the EMC [8] have been described in the framework of QCD in [9].

However, subsequent measurements at HERA (by H1) were interpreted as a practical Q^2 independence [10] of the quantity $\langle n\rangle^{\mathrm{DIS}}(W,Q^2)$ (for the current hemisphere in the hadronic c.m.s.), while H1 [12,13] and ZEUS [14,15] reported quite fast Q^2 dependence for the current hemisphere in the Breit frame. It should be noted, however, that this last result concerns different bins in W for changing Q^2 values. Anyway, the situation is controversial and therefore very interesting.

In this paper we give our own interpretation of the HERA data on charged hadron multiplicities in the current fragmentation region; as will be seen in the text below, these are in agreement with Yang's general hypothesis [1] and our early QCD results [3,4] (see also the review [11]).

2 Hadronic spectrum and multiplicity in DIS

According to the factorization for inclusive spectra in DIS, the hadronic spectrum in DIS is represented by two terms:

$$\frac{\mathrm{d}n}{\mathrm{d}y}^{\mathrm{DIS}}(W,Q^2,y) = \int_{x_0}^{1} \frac{\mathrm{d}z}{z} w(x,z,Q^2) \frac{\mathrm{d}\hat{n}}{\mathrm{d}y}(W_{\mathrm{eff}}, y-y_0) + \frac{\mathrm{d}n_0}{\mathrm{d}y}, \tag{2}$$

[a] e-mail: kisselev@mx.ihep.su
[b] e-mail: petrov@mx.ihep.su

where $x_0 = x + (1-x)(m_h/W)\exp(-y)$, y is the rapidity of the detected hadron in the c.m.s. of DIS and m_h is its mass. In (2) $\mathrm{d}\hat{n}/\mathrm{d}y$ defines the hadronic spectrum in partonic subprocess, while the quantity $\mathrm{d}n_0/\mathrm{d}y$ describes the spectrum of the proton remnant. The latter does not contribute to the current fragmentation region of DIS at HERA energies. The quantity

$$y_0 = -\frac{1}{2}\ln\left(\frac{1-x}{z-x}\right) \quad (3)$$

determines the rapidity of the center of mass of the partonic subprocess in the center of mass of the complete process.

Correspondingly, the average hadronic multiplicity in DIS is represented by

$$\langle n\rangle^{\mathrm{DIS}}(W,Q^2) = \int_{x_0}^{1}\frac{\mathrm{d}z}{z}w(x,z,Q^2)\langle\hat{n}\rangle(W_{\mathrm{eff}}) + \langle n_0\rangle. \quad (4)$$

For small x the weight $w(x,z,Q^2)$ in (2) and (4) is of the form

$$w(x,z,Q^2) = D_{\mathrm{g}}^{q}\left(\frac{x}{z},Q^2,Q_0^2\right) f_{\mathrm{g}}(z,Q_0^2) \times \left(\int_{x_0}^{1}\frac{\mathrm{d}z}{z}D_{\mathrm{g}}^{q}\left(\frac{x}{z},Q^2,Q_0^2\right) f_{\mathrm{g}}(z,Q_0^2)\right)^{-1}. \quad (5)$$

Here D_{g}^{q} is a distribution of a quark with virtuality Q^2 in a gluon with virtuality Q_0^2, while f_{g} is a distribution of the initial gluon inside the nucleon (see [11] for details). As can be seen, the hadronic spectrum in the partonic subprocess, $\mathrm{d}\hat{n}/\mathrm{d}y$, and the hadronic multiplicity $\langle\hat{n}\rangle$ depend on the effective energy, which is smaller than W:

$$W_{\mathrm{eff}}^2 = \frac{z-x}{1-x}W^2. \quad (6)$$

In what follows, we shall work in the c.m.s. of the final hadrons. In terms of the rapidity, the current region in the c.m.s. corresponds to

$$-Y < y < 0, \quad (7)$$

with $Y = \ln(W/m_h)$ (it is assumed that the proton goes in the positive direction).

In our papers [3,4] it has been established that the total hadronic multiplicity in the partonic subprocess of DIS is related to the hadronic multiplicity in e^+e^- annihilation (up to small NLO corrections, which decrease in Q^2):

$$\langle\hat{n}\rangle(\hat{W},Q^2) \simeq \langle n\rangle^{e^+e^-}(\hat{W}), \quad (8)$$

where $\hat{W}$ is the energy of the partonic subprocess.

In the partonic subprocess, the rapidity varies in the range

$$-\hat{Y} < y - y_0 < \hat{Y}. \quad (9)$$

On integration over z, the region (9) is "smeared" into the region

$$-Y < y < Y. \quad (10)$$

The average value of the effective energy in (2), (4), available for particle production, appeared to be dependent on both W and Q^2 [3,4]:

$$\langle W_{\mathrm{eff}}^2\rangle \simeq \kappa(Q^2)W^2. \quad (11)$$

The efficiency factor $\kappa(Q^2)$, which stands in front of W^2 in (11), is much less than 1 and grows slowly in Q^2.

From formulas (4), (8) and (11), one can see that the rise of the average hadronic multiplicity in DIS has the same physical nature as in e^+e^- annihilation. For the first time this behavior has been experimentally established by H1 in 1996 [10].

However, the QCD growth of $\langle n\rangle$ is delayed in DIS by the bound-state effects and the slow QCD evolution of the structure function. This is why we predicted that the Q^2 dependence of $\langle n\rangle^{\mathrm{DIS}}(W,Q^2)$ at fixed W should remain numerically weak at HERA energies [9,11].

It follows from (9) and (3) that the center of the spectrum is shifted to the region of negative rapidities and tends to zero at asymptotically high Q^2 [4]:

$$\langle y_0\rangle|_{Q^2\to\infty} \sim -\frac{1}{\ln(\ln Q^2)}. \quad (12)$$

The hadronic spectrum in partonic subprocesses in the c.m.s. of DIS has the form

$$\frac{\mathrm{d}\hat{n}}{\mathrm{d}\hat{y}}(\hat{W},\hat{y}) = n^{e^+e^-}(\hat{W})\,\bar{D}^h(\hat{W},\hat{y}), \quad (13)$$

where $\hat{y}$ is a rapidity of the detected hadron in the c.m.s. of the partonic subprocess. The QCD spectrum $\bar{D}^h$ will be described in details in Sect. 3 (see (19)–(26)). It is normalized to unity and a normalization in the r.h.s. of (13) is done in agreement with (4).

In order to calculate the average multiplicity of charged hadrons in the current fragmentation region in the c.m.s. of DIS, one has to average the spectrum (13) with the weight (5) and then to integrate it over the region (9). The resulting expression will depend on some average value of the effective energy (6), $\langle W_{\mathrm{eff}}\rangle$, not on the total energy W.

Moreover, this multiplicity should be larger than the half of the total multiplicity of charged hadrons in the partonic subprocess taken at $\langle W_{\mathrm{eff}}\rangle$ due to a shift in rapidity, y_0, in the r.h.s. of (2).

Thus, contrary to the usual opinion, there are no simple relations between the hadron multiplicity in the current fragmentation region of the c.m.s. of DIS and the hadron multiplicity in a quark jet in e^+e^- annihilation, although the underlying expression (13) is defined in terms of $\langle n\rangle^{e^+e^-}$.

3 Hadronic multiplicities in the current fragmentation region

To calculate the multiplicity of charged hadrons in the current fragmentation region, we have to define expressions of the quark distribution at small x, of the hadronic spectrum in the partonic subprocess as well as of the multiplicity of charged hadrons in e^+e^- annihilation.

For the quark distribution, we use an analytical expression from [16], in the case of soft initial conditions. As was shown in [16], at small x it is in good agreement with the data on the structure function from HERA in a wide range of Q^2. Namely, at high Q^2, we have

$$D_{\mathrm{g}}^q(z, Q^2) \sim r I_1(t) \exp(-d\xi/2), \tag{14}$$

with $d = \beta_0 + 20N_f/27$. Here I_1 is the first-order modified Bessel function of the first kind.

It is worth to note that our final results depend weakly on the very expression of the distribution D_{g}^q, provided it describes the data on the structure functions well. This is due to the fact that D_{g}^q enters both the numerator and denominator in the r.h.s. of (5). We prefer to use the expression (14) from [16], because it is presented in an analytical form.

The variable

$$t = 2\sqrt{6\xi \ln\left(\frac{1}{z}\right)} \tag{15}$$

is related to the QCD evolution parameter

$$\xi = \frac{2}{\beta_0} \ln\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right), \tag{16}$$

where $\beta_0 = 11 - 2N_f/3$ is the β-function in lowest order and

$$r = \frac{t}{2\ln(1/z)}. \tag{17}$$

The quark and gluon distributions from [16] obey the DGLAP evolution equations [17].

The expression of the initial gluon distribution at z close to 1 is chosen to have the following usually adopted form:

$$f_{\mathrm{g}}(z, Q_0^2)|_{z\to 1} \sim (1-z)^{n_{\mathrm{g}}}, \tag{18}$$

where the value of n_{g} should be correlated with a value of the parameter Q_0 (see the remark after (29)). We have omitted constant factors in the r.h.s. of (14) and (18) as they do not influence our final results.

The spectrum of hadrons in the partonic process $\bar{D}^h$ was calculated by many authors. We use the expression from [18] (N is a normalization factor):

$$\bar{D}^h(W, \zeta) = \frac{N}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k + \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right] \tag{19}$$

calculated in the variable

$$\zeta = \ln\left(\frac{W}{E_h}\right). \tag{20}$$

Here E_h is the energy of the detected hadron.

The average value of ζ, ζ_0, and its dispersion σ are given by the formulas

$$\zeta_0 = \frac{1}{2}\tau\left(1 + \frac{\rho}{24}\sqrt{\frac{48}{\beta_0\tau}}\right)\left(1 - \frac{\omega}{6\tau}\right), \tag{21}$$

$$\sigma = \sqrt{\frac{\tau}{3}}\left(\frac{\beta_0\tau}{48}\right)^{1/4}\left(1 - \frac{\beta_0}{64}\sqrt{\frac{48}{\beta_0\tau}}\right)\left(1 + \frac{\omega}{8\tau}\right), \tag{22}$$

where

$$\tau = \ln\left(\frac{W}{\Lambda}\right) \tag{23}$$

and

$$s = -\frac{\rho}{16}\sqrt{\frac{3}{\tau}}\left(\frac{48}{\beta_0\tau}\right)^{1/4}\left(1 + \frac{\omega}{4\tau}\right), \tag{24}$$

$$k = -\frac{27}{5\tau}\left(\sqrt{\frac{\beta_0\tau}{48}} - \frac{\beta_0}{24}\right)\left(1 + \frac{5\omega}{12\tau}\right), \tag{25}$$

$$\delta = \frac{\zeta - \zeta_0}{\sigma}. \tag{26}$$

Here $\rho = 11 + 2N_f/27$, $\omega = 1 + N_f/27$.

At low (effective) energies we use the fit of the low-energy data on the multiplicity of charged hadrons in e^+e^- annihilation from [19]:

$$\langle n\rangle^{e^+e^-} = 2.67 + 0.48 \ln W^2, \tag{27}$$

while for high energies ($W_{\mathrm{eff}} > 10\,\mathrm{GeV}$) we apply the fit from [20], which describes well the e^+e^- data up to LEP energies:

$$\langle n\rangle^{e^+e^-} = -1.66 + 0.866 \exp(1.047\sqrt{\ln W^2}). \tag{28}$$

We have corrected the fit (27) for a fraction of the charged particles from K_S^0 and $\Lambda(\bar{\Lambda})$ decays by a factor $1-R$, where $R = 0.097$ [21].

Let us stress the point that our formula (13) relates the hadronic spectrum with the *experimentally observable* number of charged hadrons in e^+e^- annihilation. That is why one can use any phenomenological fit of $\langle n\rangle^{e^+e^-}(W)$ which describes the data in the corresponding range of energy W.

The results of the fit of the data on charged multiplicity in the current hemisphere of the c.m.s. [10] by using formulas (2) and (13) are shown in Fig. 1. The solid curves correspond to the following values of the parameters:

$$Q_0^2 = 0.96\,\mathrm{GeV}^2, \quad \Lambda = 0.25\,\mathrm{GeV}, \quad n_{\mathrm{g}} = 6.1. \tag{29}$$

The parameter $n_{\mathrm{g}} = 6.1$ in (19) appeared to be close to the corresponding value of one of the MRST sets of parton distributions taken at $Q_0 = 1\,\mathrm{GeV}$ [22]. Note that this value is close to the value $Q_0 = 0.7\,\mathrm{GeV}$ used previously to describe the EMC data on $\langle n\rangle^{\mathrm{DIS}}(W, Q^2)$ [9].

As can be seen, our QCD predictions are quite compatible with a slow growth of $\langle n\rangle^{\mathrm{DIS}}(W, Q^2)$ in Q^2 at fixed W.

In the framework of so-called topological jet universality (based on the dual quark-line topological diagrams) there the same conclusion was obtained (see [23] and references therein). The dual parton model [6] predicts a very weak dependence on Q^2 at fixed W in the central rapidity region of DIS.

Let us note that the H1 data are in contradiction with the rapid growth of $\langle n\rangle^{\mathrm{DIS}}$ with Q^2 predicted in [5].

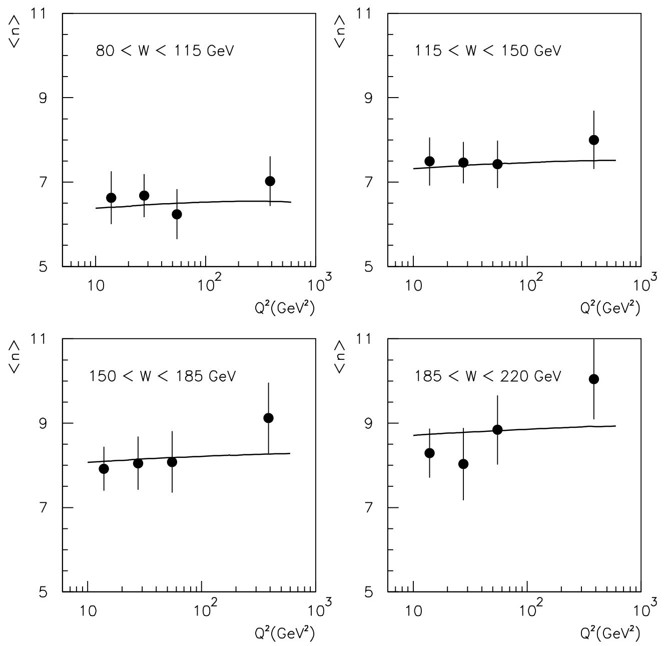

Fig. 1. The Q^2 dependence of the multiplicity of charged hadrons in the current fragmentation region of the c.m.s. in intervals of W

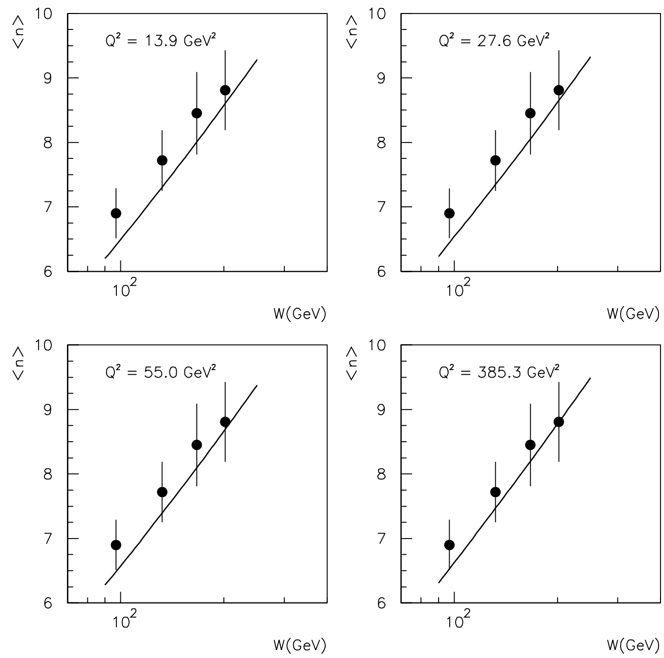

Fig. 2. The W dependence of the multiplicity of charged hadrons in the current fragmentation region of the c.m.s. at fixed values of Q^2

The Q^2 dependence is expected to remain rather weak at higher values of Q^2 an W. For example, our calculations show that at the fixed value of $W = 200$ (600) GeV the hadronic multiplicity $\langle n\rangle^{\mathrm{DIS}}(W, Q^2)$ varies from 8.70 (13.03) to 8.93 (13.31) with the increase of Q^2 from 10 (10^2) GeV^2 to $6 \cdot 10^2$ ($6 \cdot 10^3$) GeV^2.

Figure 2 demonstrates a rapid rise of $\langle n\rangle^{\mathrm{DIS}}(W, Q^2)$ in the variable W for different values of Q^2, which was predicted many years ago in [3,4] and which was seen previously in e^+e^- annihilation (the very values of Q^2 taken from [10]). Let us note that the H1 data presented in the literature (see Table 4 in [10]) do not correspond to some fixed values of Q^2, in contrast with the experimental points in Fig. 1. The curves in Fig. 2 have been calculated by using the result of our fit (29).

In order to obtain the multiplicity of charged hadrons in the current region of the Breit frame, the c.m.s. spectrum (13) has to be integrated in the region

$$-Y < y < y_{\mathrm{B}}, \tag{30}$$

where

$$y_{\mathrm{B}} = -\frac{1}{2}\ln\left(\frac{1+v}{1-v}\right) \simeq -\frac{1}{2}\ln\left(\frac{1}{x}\right). \tag{31}$$

The quantity

$$v = 1 - |1 - 2x| \tag{32}$$

in the r.h.s. of (31) is the velocity of the Breit frame in the c.m.s. So y_{B} corresponds to zero rapidity in this frame.

The results of our calculations of the multiplicity of charged hadrons in the current region of the Breit frame are presented in Fig. 3 as a function of Q^2, in comparison with the H1 data [13] (solid squares) and ZEUS data [15] (solid circles). Our prediction for $\langle n\rangle^{\mathrm{DIS}}$ in the Breit frame

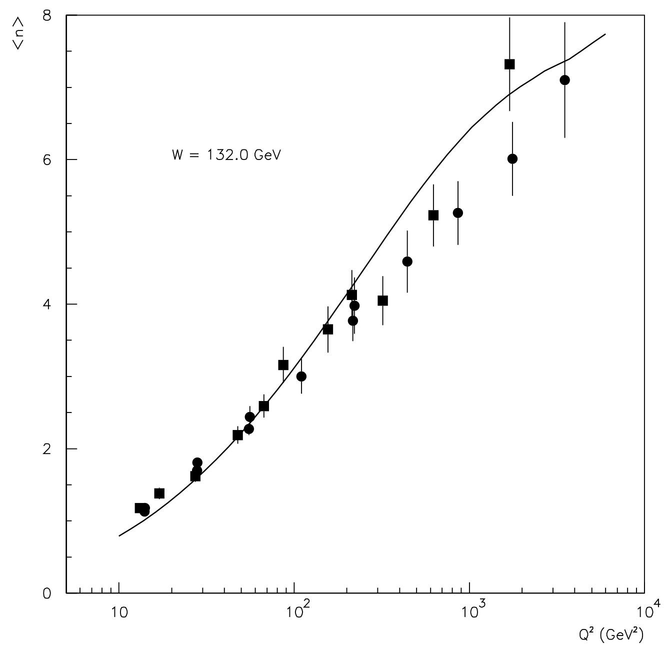

Fig. 3. The Q^2 dependence of the multiplicity of charged hadrons in the current fragmentation region of the Breit frame at fixed values of W

(solid curve in Fig. 3) has been obtained by using the same values of the parameters (29).

It should be noted that the strong Q^2 dependence seen by H1 and ZEUS in the Breit frame has nothing to do with the Q^2 dependence of $\langle n\rangle_F^{\mathrm{DIS}}(W, Q^2)$ in the c.m.s. (see Fig. 1); to a large extent it has a kinematical origin. The point is that an increase of Q^2 at fixed W is equivalent to an increase of x. As a result, the current region of the

Breit frame (30) is enlarged. Thus, the rapid growth of hadronic multiplicity in the Breit frame in Q^2 (at fixed W) reflects a strong increase of the hadronic spectrum towards the central region.

As for the increase of $\langle n \rangle_F^{\mathrm{DIS}}(x, Q^2)$ in Q^2 at fixed x in the Breit frame, it has been found that this is similar to that in e^+e^- annihilation at high Q^2, while there is a discrepancy between the DIS and the e^+e^- data at low Q^2 [12–15]. Within our approach, it can be understood as follows. On the one hand, the increase of Q^2 results in an increase of the height of the spectrum (because W grows). On the other hand, the position of the spectrum, $\langle y_0 \rangle$ as defined in (3), tends towards the region of positive rapidities.

These two phenomena go in opposite directions. At Q^2 high enough, $\langle y_0 \rangle$ varies very slowly with Q^2 (12). As a result, the rise of the forward hadronic multiplicity in the Breit frame looks similar to that in e^+e^- annihilation. At low Q^2, $\langle y_0 \rangle$ changes more significantly [4]. This effect partially compensates the growth of the spectrum and there appears a significant difference between the DIS and the e^+e^- data.

Finally, it is interesting to analyze the case when x increases while Q^2 remains fixed. In this case, W decreases, which results in a rapid decrease of the spectrum. At the same time, however, the current region in the Breit frame becomes larger in accordance with formulas (30) and (31). The effects are of the same order but opposite in sign. The ZEUS data in the Breit frame (see Table 2 in [15]) show that there is a slow rise of the hadronic multiplicity in the current hemisphere in x at different fixed values of Q^2.

To conclude, the observed rise of the hadronic multiplicity with Q^2 in the current region of the Breit frame has both a dynamical and a kinematical origin. This is why a direct comparison of the available DIS data in the Breit frame with the e^+e^- data is not completely correct.

4 Conclusions

In this paper we have presented the results of QCD calculations of the multiplicity of charged hadrons in the current hemisphere of DIS. Both the c.m.s. and the Breit frame are considered. It is demonstrated that previously derived QCD formulae successfully describe the data on the hadron multiplicities at HERA energies. The dependence of $\langle n \rangle^{\mathrm{DIS}}$ as a function of Q^2 and W (or x) is discussed in detail.

We have shown that the H1 data are in agreement with Yang's hypothesis and our QCD predictions. Namely, the efficiency of high energy collisions does weakly depend on the hard scale of the process (momentum transfer Q^2). It means that *at fixed energy* the efficiency of a particle production in hard processes increases with the shrinking of the interaction region ($\sim 1/Q$, in DIS), though in a quite slow way at the available Q^2.

New data from HERA on the hadronic multiplicity in the current region of the c.m.s. as well as measurements of the total multiplicity in DIS as functions of two variables (Q^2 and W, with higher accuracy and in a wider interval of Q^2) are badly needed to understand better the mechanism of this transformation of energy into matter.

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