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Regge-Eikonal Model for High Energy Elastic Diffractive Nucleon–Nucleon Scattering with a Minimum Number of Reggeons

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Abstract—A phenomenological Regge-eikonal model with nonlinear monotonic parameterizations for Regge trajectories in which their asymptotic behavior in the perturbative region is taken into account explicitly is proposed. It is shown with the example of elastic proton–(anti)proton scattering that in the kinematic region $\sqrt{s} >$ 23 GeV, 0.005 GeV$^2 < -t < 3$ GeV^2, the diffraction pattern is mainly determined by only three Regge trajectories.

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INTRODUCTION

The objective of this work is to demonstrate, using the example of elastic proton–(anti)proton scattering, the advantages (in the framework of the Regge-eikonal approach) of an explicit account of the asymptotic behavior of Regge trajectories for phenomenological description of elastic diffraction at high energies that follows from QCD.

In our considerations, it will be assumed that QCD is the fundamental theory of strong interaction and that Regge trajectories are invariant with respect to the renormalization group; these assumptions are justified by observed bound states and resonances. It will be shown below that in this case, due to the asymptotic freedom, Regge trajectories tend to a constant for asymptotically large values of transferred momentum.

Although in modern literature the assumption of the linear character of Regge trajectories for small negative values of the argument is postulated, the unique essential argument in favor of this statement of the problem (along with the natural desire to continue Chew–Frautchi plots to the scattering region) is the result of extraction in the framework of the Born approximation of the degenerate trajectory ρ/a_2 from data on the exchange processes $\pi^- + p \longrightarrow \pi^0 + n$ and $\pi^- + p \longrightarrow \eta + n$ [1]. However, it follows from analysis of these data that the validity of the Born approximation (i.e., the capability of neglecting absorptive corrections) is not well grounded [2]. If absorptive corrections are taken into account, the procedure of extraction of Regge trajectories from angular distributions becomes so complicated that any results obtained from available experimental data are far from unambiguous.

Therefore, from the phenomenological point of view, approaches using the linear Regge trajectories for the description of diffractive processes do not have any advantage over approaches using nonlinear parameterizations.

In the case of the approximation of Regge trajectories by monotonic functions in which their asymptotic behavior is explicitly taken into account, we obtain two important advantages over the application of linear parameterizations:

(1) difficulties related to possible occurrence of nonphysical singularities in signature factors are naturally avoided;

(2) it is possible to reproduce with sufficiently good accuracy experimentally observed diffraction pattern of elastic scattering of nucleons at collision energies $\sqrt{s} >$ 23 GeV in the framework of the minimal phenomenological scheme using only three Regge trajectories.

Upon description of elastic diffraction, the Regge-eikonal model will be used; the advantage of this model as compared to the simple Regge approach is that absorptive corrections are taken into account automatically, which explicitly satisfies the unitarity condition for the scattering amplitude [3]. Therefore (for closeness of this presentation), before discussing in detail the asymptotic properties of Regge trajectories, we will show how these trajectories are obtained in the framework of the Regge-eikonal approach.

BASICS OF THE REGGE-EIKONAL MODEL

The combination of the eikonal representation and the Regge poles was first considered in [3]. We will limit ourselves to a brief presentation of basic propositions and results.

Let us consider the process of elastic or exchange interaction of two elementary scalar particles (below, upon analysis of data on elastic nucleon–nucleon scattering, spin effects will be neglected), hereinafter conventionally called "hadrons." The main physical characteristic of this reaction is the differential scattering cross section $\frac{d\sigma}{dt}$, which can be expressed in terms of the elastic scattering amplitude $T(s, t)$ (here, s is the squared collision energy in the center-of-mass system, and t is the four-momentum transfer squared) in the limit of high energies as follows:

$$\frac{d\sigma}{dt} = \frac{|T(s,t)|^2}{16\pi s^2}. \qquad (1)$$

In the case of short-range forces, the eikonal representation of the scattering amplitude can be introduced,

$$T(s,b) = \frac{e^{2i\delta(s,b)}-1}{2i}. \qquad (2)$$

This formula (which is in essence the definition of the eikonal) is written in the coordinate representation. The Fourier–Bessel transform can be used to go from one representation to the other,

$$f(b) = \frac{1}{16\pi s}\int_0^\infty d(-t)J_0(b\sqrt{-t})f(t),$$
$$f(t) = 4\pi s\int_0^\infty db^2 J_0(b\sqrt{-t})f(b). \qquad (3)$$

The eikonal representation does not yield any progress in consideration of our problem, since it is reduced to the replacement of $T(s, t)$ by $\delta(s, t)$ without specification of the eikonal. The assumption that the eikonal is proportional with high accuracy to the "effective" (quasi-)potential of hadronic interaction is the key one, similar to that in nonrelativistic quantum mechanics, except for the fact that the (quasi-)potential is relativistic in this case. According to the Van Hove interpretation [4] of the relativistic (quasi-)potential as the "sum" of all single-particle exchanges in the t-channel, the eikonal can be represented in the form

$$\delta^{(f_1,f_2)}(s,t) = \sum_{j=0}^{\infty}\sum_{m_j} J^{(f_1,j,m_j)}_{\alpha_1\ldots\alpha_j}(p_1,\Delta)\frac{D^{\alpha_1\ldots\alpha_j,\beta_1\ldots\beta_j}_{(j,m_j)}}{m_j^2-\Delta^2}J^{(f_2,j,m_j)}_{\beta_1\ldots\beta_j}(p_2,\Delta). \qquad (4)$$

Here, $\frac{D^{\alpha_1\ldots\alpha_j,\beta_1\ldots\beta_j}_{(j,m_j)}}{m_j^2-\Delta^2}$ is the propagator of the particle with the spin j and the mass m_j, $J^{(f,j,m_j)}_{\alpha_1\ldots\alpha_j}$ is the hadronic current (the index f characterizes the current type), Δ is the transferred four-momentum ($t = \Delta^2$), p_1 and p_2 are the four-momenta of incident hadrons, and the sign $\sum_{m_j}$ denotes summing over all particles with the spin j and different masses, which is further transformed into the sum over Regge trajectories. We impose the following constraints on the general dependence of all possible hadronic currents on Δ: complete symmetry with respect to all α_k, transversality with respect to Δ_{α_k} ($k = 1, \ldots, j$), and tracelessness with respect to any pair of indices. The first two conditions yield

$$J^{(f,j,m_j)}_{\alpha_1\ldots\alpha_j}(p,\Delta) = \sum_{k=0}^{\left[\frac{j}{2}\right]}\Gamma_k^{(f,j,m_j)}(p^2,\Delta^2,(p\Delta))$$
$$\times\sum G_{\alpha_{\mu_1}\alpha_{\mu_2}}\ldots G_{\alpha_{\mu_{2k-1}}\alpha_{\mu_{2k}}}P_{\alpha_{\mu_{k+1}}}\ldots P_{\alpha_{\mu_j}},$$

where $\Gamma_k^{(f,j,m_j)}(p^2, \Delta^2, (p\Delta))$ are some scalar functions,

$P_\alpha \equiv \frac{p_\alpha - \frac{p\Delta}{\Delta^2}\Delta_\alpha}{\sqrt{p^2-\frac{(p\Delta)^2}{\Delta^2}}}$, $G_{\alpha\beta} \equiv g_{\alpha\beta} - \frac{\Delta_\alpha\Delta_\beta}{\Delta^2}$, and the inner summing is carried out over all nonequivalent permutations of Lorentz indices (the total of $\frac{j!}{(2k)!!(j-2k)!}$ terms). The condition of tracelessness, taking into account that

$$g^{\alpha\beta}G_{\alpha\beta} = -3, \quad P_\alpha P^\alpha = 1, \quad g^{\gamma\delta}G_{\alpha\gamma}G_{\beta\delta} = -G_{\gamma\delta},$$
$$G_{\alpha\gamma}P^\beta = P^\alpha,$$

results in the recurrent relations

$$\Gamma_k^{(f,j,m_j)}(p^2,\Delta^2,(p\Delta)) = \frac{\Gamma_{k-1}^{(f,j,m_j)}(p^2,\Delta^2,(p\Delta))}{2(j-k)+1}.$$

From this we find

$$J^{(f,j,m_j)}_{\alpha_1\ldots\alpha_j}(p,\Delta)$$
$$= \Gamma_0^{(f,j,m_j)}(p^2,\Delta^2,(p\Delta))\sum_{k=0}^{\left[\frac{j}{2}\right]}\frac{(2(j-k)-1)!!}{(2j-1)!!} \qquad (5)$$
$$\times\sum G_{\alpha_{\mu_1}\alpha_{\mu_2}}\ldots G_{\alpha_{\mu_{2k-1}}\alpha_{\mu_{2k}}}P_{\alpha_{\mu_{k+1}}}\ldots P_{\alpha_{\mu_j}}.$$

By substituting (5) into (4) and taking into account the transversality and the tracelessness of hadronic currents, we obtain the following expression for the eikonal:

$$\delta^{(f_1, f_2)}(s, t) = \sum_{j=0}^{\infty}\sum_{m_j}\frac{\gamma^{(f_1, f_2, j, m_j)}(\Delta^2)}{m_j^2 - \Delta^2}$$
$$\times P_j\left(\frac{p_1 p_2 - \dfrac{(p_1\Delta)(p_2\Delta)}{\Delta^2}}{\sqrt{\left(p_1^2 - \dfrac{(p_1\Delta)^2}{\Delta^2}\right)\left(p_2^2 - \dfrac{(p_2\Delta)^2}{\Delta^2}\right)}}\right),$$

where $P_j(x)$ are the Legendre polynomials of degree j and

$$\gamma^{(f_1, f_2, j, m_j)}(\Delta^2) \equiv \Gamma_0^{(f_1, j, m_j)}(p_1^2, \Delta^2, (p_1\Delta))$$
$$\times \Gamma_0^{(f_2, j, m_j)}(p_2^2, \Delta^2, (p_2\Delta))\frac{2^j (j!)^2}{(2j)!}.$$

For similar external particles on the mass shell,

$$p_{1,2}^2 = (p_1 - \Delta)^2 = (p_2 + \Delta)^2 = m^2. \quad (6)$$

By applying the kinematic relations

$$p_{1,2}\Delta = \pm\frac{\Delta^2}{2}, \quad p_1 p_2 = \frac{s}{2} - m^2, \quad \Delta^2 \equiv t,$$

we simplify the expression for the eikonal (from now on, the indices f_1 and f_2 characterizing the type of single-particle exchange will be omitted),

$$\delta(s, t) = \sum_{j=0}^{\infty}\sum_{m_j}\frac{\gamma^{(j, m_j)}(t)}{m_j^2 - t}P_j\left(\frac{s}{2m^2 - \dfrac{t}{2}} - 1\right). \quad (7)$$

We divide the sum over all j in the right-hand side of (7) into two sums over even and odd j,

$$\delta(s, t) = \delta^+(s, t) + \delta^-(s, t)$$
$$= \sum_{j=0}^{\infty}\sum_{\eta=\pm 1}\sum_{m_{(\eta)j}}\frac{\eta + e^{-i\pi j}}{2}(-1)^j\frac{\gamma^{(\eta, j, m_{(\eta)j})}(t)}{m_{(\eta)j}^2 - t} \quad (8)$$
$$\times P_j\left(\frac{s}{2m^2 - \dfrac{t}{2}} - 1\right).$$

Each of the sequences $\gamma^{(\eta, j, m_{(\eta)j})}(t)$, $m_{(\eta)j}^2$ ($j = 0, 2, 4, \ldots, 2n$, for $\eta = +1$ and $j = 1, 3, 5, \ldots, 2n - 1$, for $\eta = -1$) separately satisfies the condition of Carlson's theorem, which states that if for the function $f(x)$ of a complex variable the condition $f(x) < e^{\pi|x|}$ is satisfied for $|x| \longrightarrow \infty$, this function is uniquely determined by its values for integer x.

We make the key assumption on the possibility of a unique (due to Carlson's theorem) analytic continuation of (8) to the region of complex values of j (the Regge hypothesis or the postulate of maximum analyticity of the second order). In our case, it is reduced to the proposition that $m_{(\eta)j}^2$ and $\gamma^{(\eta, j, m_{(\eta)j})}(t)$ in (7) are the values of analytic (holomorphic with respect to the complex variable j) functions for integer nonnegative values of j. We denote these functions by $m_\eta^2(j)$ and $\gamma^{(\eta)}(t, j, m_\eta^2(j))$, respectively. Using the Sommerfeld–Watson transform [5], we replace the sum over j in (8) by the integral over the contour C on the complex plane of the variable j encircling the real positive half-axis including the point $j = 0$ in such a way that the half-axis is on the right,

$$\delta(s, t) = \frac{1}{2i}\oint_C\frac{dj}{\sin(\pi j)}\sum_{\eta=\pm 1}\sum_{m_\eta}\left(-\frac{\eta + e^{-i\pi j}}{2}\right)$$
$$\times\frac{\gamma^{(\eta)}(t, j, m_\eta^2(j))}{m_\eta^2(j) - t}P\left(j, \frac{s}{2m^2 - \dfrac{t}{2}} - 1\right).$$

Here, $P(j, x)$ is the analytic continuation of Legendre polynomials $P_j(x)$ to the region of complex j.

Since (according to our assumption) the unique sources of singularities of the integrand in the region $\mathrm{Re}j > -1/2$ are the zeros of the functions $\sin(\pi j)$ and $m_\eta^2(j) - t$, by deforming the contour C and passing to the contour parallel to the imaginary axis, $\mathrm{Re}j = -1/2$, we obtain

$$\delta(s, t) = \frac{1}{2i}\int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty}\frac{dj}{\sin(\pi j)}\sum_{\eta=\pm 1}\sum_{m_\eta}\left(-\frac{\eta + e^{-i\pi j}}{2}\right)$$
$$\times\frac{\gamma^{(\eta)}(t, j, m_\eta^2(j))}{m_\eta^2(j) - t}P\left(j, \frac{s}{2m^2 - \dfrac{t}{2}} - 1\right)$$
$$+\sum_{\eta=\pm 1}\sum_n\left(-\frac{\eta + e^{-i\pi\alpha_n^{(\eta)}(t)}}{\sin(\pi\alpha_n^{(\eta)}(t))}\right)\frac{d\alpha_n^{(\eta)}(t)}{dt}$$
$$\times\frac{\pi\gamma^{(\eta)}(t, \alpha_n^{(\eta)}(t), t)}{2}P\left(\alpha_n^{(\eta)}(t), \frac{s}{2m^2 - \dfrac{t}{2}} - 1\right),$$

where the functions $\alpha_n^{(\eta)}(t)$ are the roots of the equations $m_\eta^2(j) - t = 0$ and thus correspond to the poles of the eikonal in the region of complex j. These poles are called Regge poles, and the functions $\alpha_n^{(\eta)}(t)$ are called Regge trajectories (C-even for $\eta = +1$ and C-odd for $\eta = -1$). For $s \gg 2m^2 - t/2$, the contribution of the background integral can be neglected, and the Legendre polynomials can be described by the leading terms of the expansion. Therefore, by introducing the new function

$$\beta_n^{(\eta, s_0)}(t) \equiv \frac{d\alpha_n^{(\eta)}(t)}{dt}\frac{\pi\gamma^{(\eta)}(t, \alpha_n^{(\eta)}(t), t)}{2} \times \frac{\Gamma(2\alpha_n^{(\eta)}(t)+1)}{2^{\alpha_n^{(\eta)}(t)}\Gamma^2(\alpha_n^{(\eta)}(t)+1)}\left(\frac{s_0}{2m^2 - \frac{t}{2}}\right)^{\alpha_n^{(\eta)}(t)},$$

where s_0 is any scale determined a priori (for example, $s_0 = 1$ GeV^2), and $\Gamma(x)$ is the Euler gamma function (note that the function $\beta_n^{(\eta, s_0)}(t)$ depends on the chosen scale s_0; from now on, the index s_0 in the notation of this function will be omitted), we obtain in the limit of high energies

$$\delta(s, t) = \sum_n \left(i + \tan\frac{\pi(\alpha_n^+(t)-1)}{2}\right)\beta_n^+(t)\left(\frac{s}{s_0}\right)^{\alpha_n^+(t)} + \sum_n \left(i - \cot\frac{\pi(\alpha_n^-(t)-1)}{2}\right)\beta_n^-(t)\left(\frac{s}{s_0}\right)^{\alpha_n^-(t)}. \quad (9)$$

This formula (note that it is valid for those single-particle exchanges for which (6) is violated), together with the eikonal representation of amplitude (2), is the essence of the Regge-eikonal model.

Thus, the practical advantage of the Regge-eikonal approach is that it is possible to decrease the functional arbitrariness by reducing the unknown function of two variables $T(s, t)$ to several functions of one variable, which are Regge trajectories and Regge residues.

BEHAVIOR OF REGGE TRAJECTORIES IN THE REGION OF REAL NEGATIVE VALUES OF THE ARGUMENT

It was already noted above that the unique strictly established constraints on the functional form of Regge trajectories in the region of negative values of the argument are their real character and the invariance with respect to the renormalization group. To obtain more detailed information on these functions, it is necessary to consider particular quantum field models.

In the case of QCD, any function of one dynamical variable invariant with respect to the renormalization group can be represented in the region of asymptotically large negative values of this variable (when mass effects of quark fields can be neglected) in the form of the general solution to the Ovsyannikov equation [6],

$$f(t) \equiv \tilde{f}\left(\frac{t}{\mu^2}, \alpha_s(\mu)\right) = \Phi\left(\frac{t}{\mu^2}e^{K(\alpha_s(\mu))}\right),$$

where $\Phi(x)$ is some analytic function, μ is the dimensional renormalization parameter, $\alpha_s(\mu) \equiv g_s^2(\mu)/4\pi$ is the running coupling constant, and $K'(\alpha_s) = 1/\beta(\alpha_s) \equiv \left(\mu^2\frac{\partial\alpha_s}{\partial\mu^2}\right)^{-1}$ (due to the asymptotic freedom, $K(\alpha_s(\mu) \longrightarrow -\infty$ for $\mu \longrightarrow +\infty$). By passing to the limit $\mu = \sqrt{-t}$, $t \longrightarrow -\infty$, we obtain

$$\lim_{t \to -\infty} f(t) = \Phi(0) = \text{const}. \quad (10)$$

In the potential scattering theory, the slope of the Regge trajectory $\alpha(t)$ is related to the squared effective radius of the interaction, generating this trajectory approximately as $R^2 \sim \alpha'(t)(2\alpha(t)+1)$ [5, 7]. Therefore, in the quantum mechanical sense, $\lim_{t\to -\infty}\alpha(t) = \text{const}$ indeed corresponds to small distances and the limit of free fields for asymptotically large values of the transferred momentum (note that this correspondence is absent for $\alpha(t) = \alpha_0 + \alpha'_0 t$, $\alpha'_0 > 0$, when $R^2 \longrightarrow -\infty$ for $t \longrightarrow -\infty$).

Information concerning the behavior of particular Regge trajectories additional to (10) can be obtained from quantitative estimates in the domain of applicability of perturbative methods (note that the application of perturbation theory is justified already for $-t > 6$ GeV^2, since the running coupling constant for these scales $\alpha_s(\sqrt{-t}) < 0.3$ [8]). Thus, for trajectories corresponding to "quark–antiquark" pair exchanges in the perturbative region, it was found [9] that

$$\alpha_{\bar{q}q}(t) = \sqrt{\frac{8}{3\pi}\alpha_s(\sqrt{-t}) + o(\alpha_s^{1/2}(\sqrt{-t}))}. \quad (11)$$

For trajectories corresponding to multigluon exchanges in the region of asymptotically large values of the transferred momentum, the following limiting relation [10–12] is satisfied:

$$\lim_{t\to -\infty}\alpha_{gg\ldots g}(t) = 1; \quad (12)$$

in particular, for the set of trajectories corresponding to the exchange of two gluons in the perturbative region [13],

$$\alpha_{gg}^{(r)}(t) = 1 + \frac{12\ln 2}{\pi}\alpha_s(\sqrt{-t}) \times \left[1 - \alpha_s^{2/3}(\sqrt{-t})\left(\frac{7\zeta(3)}{2\ln 2}\right)^{1/3}\left(\frac{r + 3/4}{11 - 2/3n_f}\right)^{2/3} + o(\alpha_s^{2/3}(\sqrt{-t}))\right], \quad (13)$$

where $\zeta(x)$ is the Riemann zeta function, n_f is the number of quark flavors, and r is the index, taking values 0, 1, 2, ….

Table 1. Results of fitting free phenomenological parameters

	Pomeron		f_2/a_2 Reggeon		ω/ρ Reggeon
p_1	0.123	c_+	0.1	c_-	0.9
p_2	1.58 GeV^{-2}				
p_3	0.15				
B_P	43.5	B_+	153	B_-	46
b_P	2.4 GeV^{-2}	b_+	4.7 GeV^{-2}	b_-	5.6 GeV^{-2}
d_1	0.43 GeV^{-2}				
d_2	0.39 GeV^{-4}				
d_3	0.051 GeV^{-6}				
d_4	0.035 GeV^{-8}				
$\alpha_P(0)$	1.123	$\alpha_+(0)$	0.78	$\alpha_-(0)$	0.64
$\alpha'_P(0)$	0.28 GeV^{-2}	$\alpha'_+(0)$	0.63 GeV^{-2}	$\alpha'_-(0)$	0.07 GeV^{-2}

At present, all practically useful QCD predictions concerning the behavior of leading meson Regge trajectories in the scattering region are limited by relations (11)–(13); therefore, for description of diffractive processes, it is necessary to construct phenomenological schemes. In this case, in each well-posed phenomenological model using the Regge approach, the applicability of relations (11)–(13) in the perturbative region should be taken into account.

The obvious corollary of (11)–(13) is the fundamental nonlinearity of Regge trajectories, although the relations do not impose any constraints on the functional form of Regge trajectories for sufficiently small t. To obtain these constraints, we apply some natural assumptions concerning the behavior of the imaginary part of leading trajectories on the physical sheet.

Any Regge trajectory $\alpha(t)$ is a real analytic function on the complex plane with a cut along the half line $(t_T, +\infty)$, $t_T > 0$ [5]; it will be assumed that $\mathrm{Im}\,\alpha(t + i0)$ increases sufficiently slowly for $t \longrightarrow +\infty$ (for example, not faster than $Ct\ln^{-1-\epsilon}t$, $\epsilon > 0$), and therefore, the following dispersion relations with not more than one subtraction will be satisfied for $\alpha(t)$ on the physical sheet:

$$\alpha(t) = \alpha_0 + \frac{t}{\pi}\int_{t_T}^{+\infty}\frac{\mathrm{Im}\,\alpha(t' + i0)}{t'(t' - t)}dt'.$$

It will also be assumed that $\mathrm{Im}\,\alpha(t + i0) \geq 0$ for $t \geq t_T$. Under these conditions (note that in the general case, they are not strictly proved; however, they are explicitly satisfied in potential scattering theory and perturbation theory [5]), $\alpha(t)$ is necessarily the Herglotz function [5]; i.e.,

$$\frac{d^n\alpha(t)}{dt^n} > 0 \quad (t < t_T, n = 1, 2, 3, \ldots). \tag{14}$$

Further, it will be assumed that relations (14) are satisfied for true Regge trajectories, which make leading contributions into the diffractive scattering amplitudes. Nonetheless, for the phenomenological description of elastic diffraction at high energies, any functions monotonic with one or several first derivatives that assume the capability of sufficiently smooth matching within the perturbative region can be used as purely quantitative approximations to true Regge trajectories in the diffractive scattering region, along with Herglotz functions.

In this regard, it should be noted that the leading trajectory in (13), $\alpha_{gg}^{(0)}(t)$, cannot be monotonically matched with the trajectory $\alpha_P(t)$, which makes the main contribution to the eikonal in the diffractive region (usually, $\alpha_P(t)$ is called the "soft" Pomeron) due to the phenomenological constraint $\alpha_P(0) < 1.15$, while $\alpha_{gg}^{(0)}(-M_Z^2) \approx 1.28$, with $M_Z \approx 91.2$ GeV. Therefore, it will be assumed from now on that the trajectory $\alpha_{gg}^{(0)}(t)$ corresponds to the so called "hard" Pomeron with the intersection $\alpha_{gg}^{(0)}(t) > 1.3$ [14], and the "soft" Pomeron exchange corresponds to the exchange by more than two gluons for asymptotically large momentum transfer; the unique constraints on the functional form of the phenomenological parameterizations of the trajectories of the "soft" Pomeron are the asymptotic condition (12) and the monotonicity assumption.

Unlike the Pomeron trajectory, for secondary trajectories corresponding to the families of observed meson

Table 2. Quality of description of experimental angular distributions

Data	Number of points	χ^2
$\sqrt{s}$ = 23 GeV (pp)	124	280
$\sqrt{s}$ = 31 GeV (pp)	154	467
$\sqrt{s}$ = 53 GeV (pp)	85	423
$\sqrt{s}$ = 62 GeV (pp)	107	409
$\sqrt{s}$ = 31 GeV ($\bar{p}p$)	38	108
$\sqrt{s}$ = 53 GeV ($\bar{p}p$)	60	336
$\sqrt{s}$ = 62 GeV ($\bar{p}p$)	40	156
$\sqrt{s}$ = 546 GeV ($\bar{p}p$)	181	352
$\sqrt{s}$ = 630 GeV ($\bar{p}p$)	19	78
$\sqrt{s}$ = 1800 GeV ($\bar{p}p$)	50	129
Total	858	2738

resonances it is possible to select the Herglotz functions that have asymptotic (11) and demonstrate reasonable (from the phenomenological point of view) behavior in the diffractive scattering region.

Note also that satisfaction of conditions (11), (12), and (14) automatically results in the absence of nonphysical singularities in signature factors in the scattering region and corresponding problem.

PARAMETERIZATION OF THE MINIMAL EIKONAL

Along with the "soft" Pomeron, there exist at least four secondary meson trajectories making a noticeable contribution into eikonal (9), namely, C-even Reggeons f_2 and a_2 and C-odd Reggeons ω and ρ. The trajectories of the secondary Reggeons with the same signature will be assumed to be approximately coinciding due to the isospin symmetry of quark flavors; i.e., $\alpha_{a_2}(t) \approx \alpha_{f_2}(t) \approx \alpha_+(t)$, and $\alpha_\rho(t) \approx \alpha_\omega(t) \approx \alpha_-(t)$. Therefore, the minimal eikonal has the form ($s_0 \equiv 1$ GeV^2)

$$\begin{aligned}\delta(s,t) &= \delta_P(s,t) + \delta_+(s,t) \mp \delta_-(s,t) \\ &= \left(i + \tan\frac{\pi(\alpha_P(t)-1)}{2}\right)\beta_P(t)\left(\frac{s}{s_0}\right)^{\alpha_P(t)} \\ &+ \left(i + \tan\frac{\pi(\alpha_+(t)-1)}{2}\right)\beta_+(t)\left(\frac{s}{s_0}\right)^{\alpha_+(t)} \\ &\mp \left(i - \cot\frac{\pi(\alpha_-(t)-1)}{2}\right)\beta_-(t)\left(\frac{s}{s_0}\right)^{\alpha_-(t)},\end{aligned} \quad (15)$$

where $\alpha_P(t)$, $\alpha_+(t)$, and $\alpha_-(t)$ are the trajectories of the Pomeron and the secondary Reggeons, $\beta_P(t)$ is the residue of the Pomeron, $\beta_+(t) \equiv \beta_{f_2}(t) + \beta_{a_2}(t)$ and $\beta_-(t) \equiv \beta_\omega(t) + \beta_\rho(t)$ are the sums of residues of C-even and C-odd secondary trajectories, respectively, and the negative (positive) sign before the C-odd contribution corresponds to the particle scattering on the particle (antiparticle).

The phenomenological parameterization for the contribution of the Pomeron into the eikonal is chosen in the form

Table 3. Quality of description of data on elastic nucleon–nucleon scattering in other sources

Reference	χ^2/d.o.f.	Kinematic region
[19]	No data	23 GeV ≤ $\sqrt{s}$ ≤ 546 GeV, 0 GeV^2 < $-t$ ≤ 4 GeV^2
[20]	2.0	53 GeV ≤ $\sqrt{s}$ ≤ 630 GeV, 0 GeV^2 < $-t$ ≤ 5 GeV^2
[21]	2.4	19 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV, 0.1 GeV^2 ≤ $-t$ ≤ 14 GeV^2
[22]	2.6	23 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV, 0.01 GeV^2 ≤ $-t$ ≤ 14 GeV^2
[23]	4.3	23 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV, 0 GeV^2 < $-t$ ≤ 6 GeV^2
[24]	2.8	23 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV, 0.01 GeV^2 ≤ $-t$ ≤ 14 GeV^2
[25]	1.5	6 GeV ≤ $\sqrt{s}$ ≤ 1800 GeV, 0.1 GeV^2 ≤ $-t$ ≤ 6 GeV^2

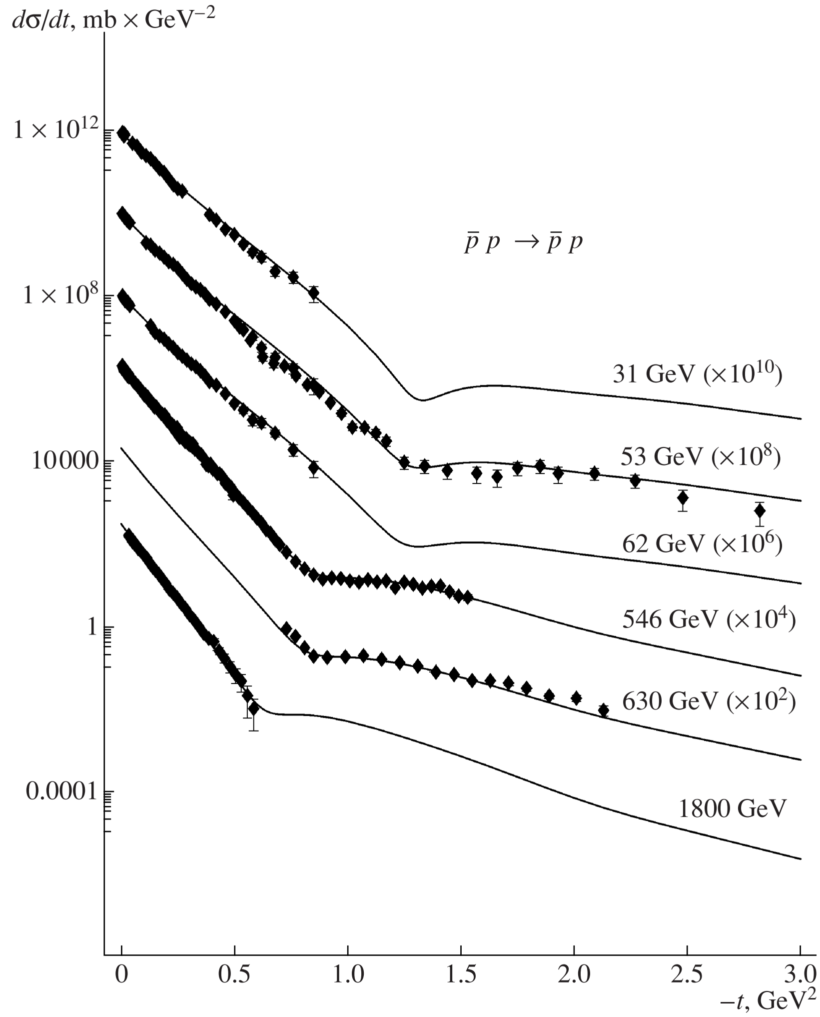

Fig. 1. Differential cross sections for the process $\bar{p}p \longrightarrow \bar{p}p$ at various collision energies.

$$\alpha_P(t) = 1 + p_1\left[1 - p_2 t\left(\arctan(p_3 - p_2 t) - \frac{\pi}{2}\right)\right], \tag{16}$$
$$\beta_P(t) = B_P e^{b_P t}(1 + d_1 t + d_2 t^2 + d_3 t^3 + d_4 t^4).$$

This approximation for the Pomeron trajectory $\alpha_P(t)$ is monotonic, and for it, asymptotic condition (12) is explicitly satisfied. The contributions of the secondary Reggeons are parameterized by the functions

$$\alpha_\pm(t) = \left(\frac{8}{3\pi}\alpha_s(\sqrt{-t + c_\pm})\right)^{1/2}, \tag{17}$$
$$\beta_\pm(t) = B_\pm e^{b_\pm t},$$

where

$$\alpha_s(\mu) = \frac{4\pi}{11 - \frac{2}{3}n_f}\left(\frac{1}{\ln\frac{\mu^2}{\Lambda^2}} + \frac{1}{1 - \frac{\mu^2}{\Lambda^2}}\right) \tag{18}$$

is the so called single-loop analytical running coupling constant [15], $n_f = 3$ is the number of quark flavors taken into account, $\Lambda \equiv \Lambda^{(3)} = 0.346$ GeV is the dimensional QCD parameter (the value is taken from [8]), and the phenomenological parameters c_+, $c_- > 0$ are sufficiently small and do not spoil asymptotic behavior (11) of secondary trajectories in the perturbative region.

Note that even for asymptotically large values of the transferred momentum, the behavior of the employed approximation (16) of the "soft" Pomeron trajectory has explicitly nonperturbative character,

$$\alpha_P(t) - 1 \sim \frac{1}{-t} \sim e^{-\frac{\text{const}}{\alpha_s(\sqrt{-t})}}, \quad (t \longrightarrow -\infty),$$

which is qualitatively different from the perturbative behavior of secondary trajectories (11).

Note also that the contribution into the eikonal of the trajectory called the "odderon," the C-odd partner of the Pomeron, was neglected. Although the validity of this neglect for small scattering angles is phenomenologi-

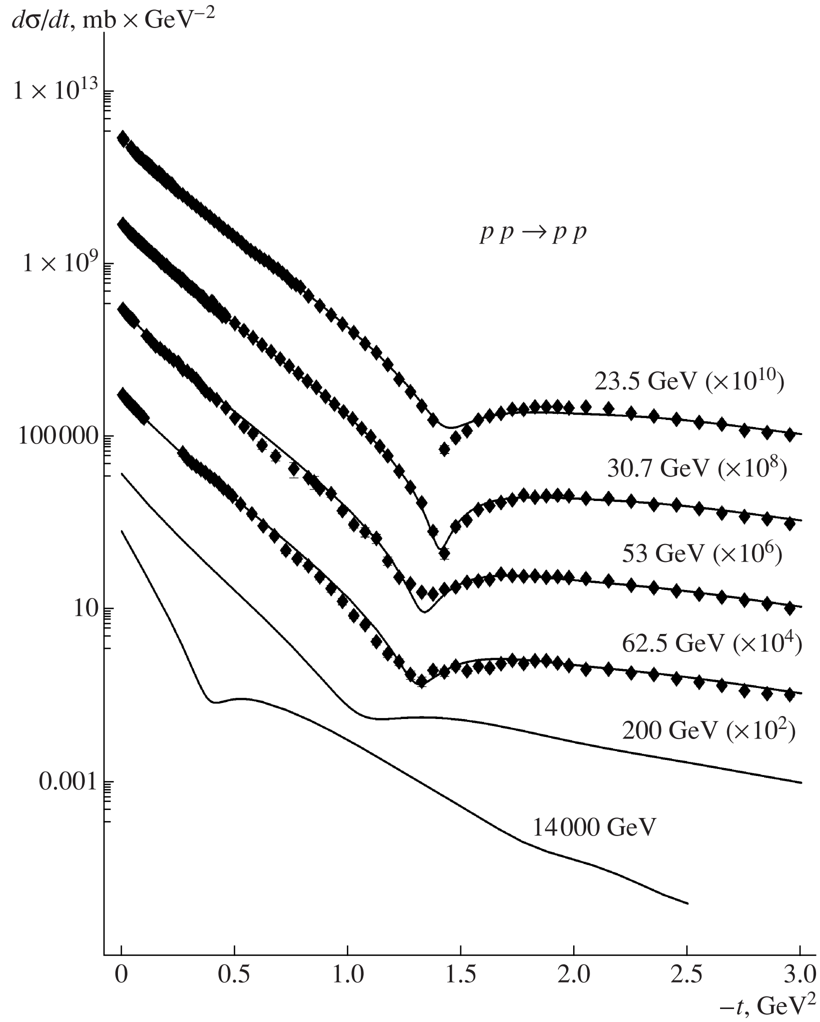

Fig. 2. Differential cross sections for the process $pp \longrightarrow pp$ at various collision energies.

cally grounded by the approximate equality of the total cross sections of proton–proton and proton–antiproton scattering at collision energies $\sqrt{s} > 200$ GeV, the adequacy of this approximation for scattering angles $-t < 3$ GeV^2 can be verified only if angular distributions in the considered kinematic region are available for both of the reactions $p + p \longrightarrow p + p$ and $\bar{p} + p \longrightarrow \bar{p} + p$. Unfortunately, at present, data on elastic diffraction for the process $p + p \longrightarrow p + p$ at energies $\sqrt{s} > 62.5$ GeV are absent. Nonetheless, neglecting the contribution of the odderon into the eikonal (15) is dictated by our desire to construct the minimal phenomenological scheme describing the diffraction pattern at high energies.

DESCRIPTION OF EXPERIMENTAL DATA

Let us describe elastic diffraction at high energies. The unique reactions measured in the sufficiently wide energy range are the processes $p + p \longrightarrow p + p$ and $\bar{p} + p \longrightarrow \bar{p} + p$. The results of fitting with respect to angular distributions in the kinematic region $\sqrt{s} > 23$ GeV, 0.005 GeV$^2 < -t < 3$ GeV^2 (at larger scattering angles, it is necessary to take into account the contributions of the so called "hard" Pomerons and odderons which make the dominant contribution into the eikonal in the perturbative region) [16][1] are presented in Tables 1 and 2 and Figs. 1 and 2.

Figures 3 and 4 show the predictions, respectively, for the total scattering cross section and the ratio of the real part of the amplitude of forward scattering and the imaginary part as functions of collision energy. In particular, for the colliders RHIC and LHC, we have $\sigma_{tot}(200\text{ GeV}) \approx 52$ mb and $\sigma_{tot}(14\text{ TeV}) \approx 111$ mb, respectively.

[1] For calculation of electromagnetic corrections to the scattering amplitude, we used the method from [17]; see also [18].

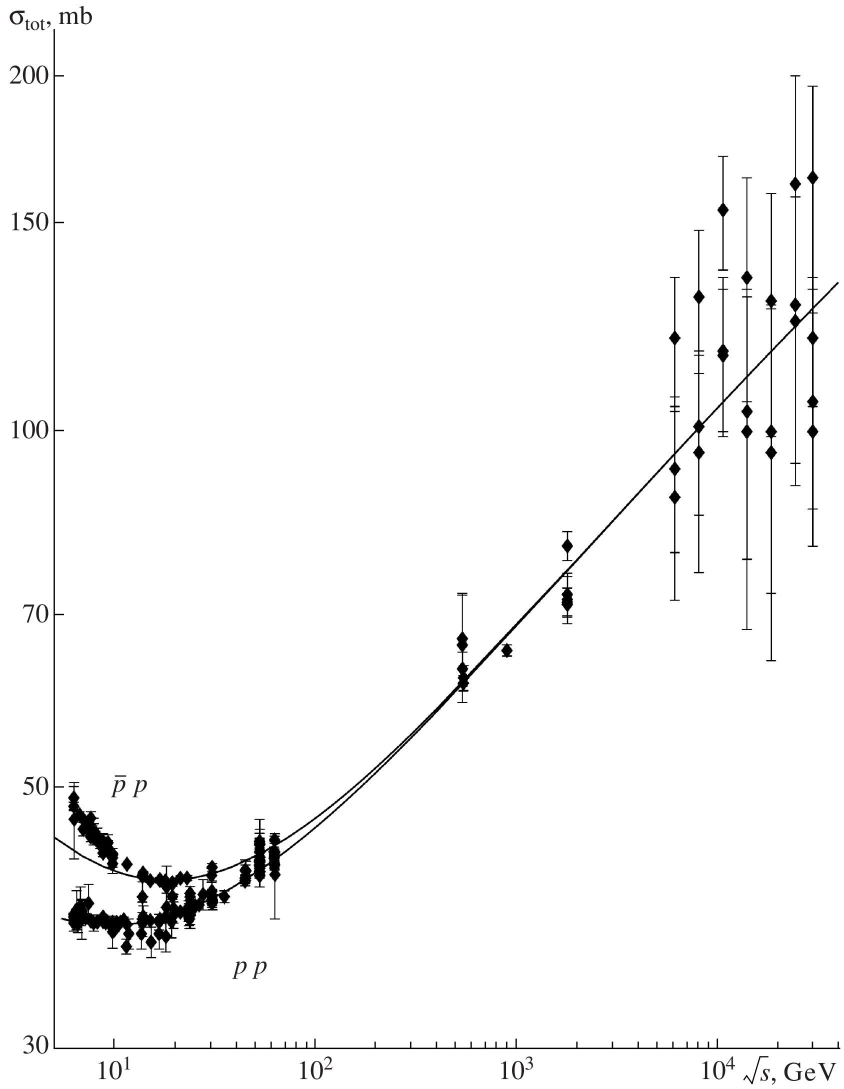

Fig. 3. Total cross section of nucleon–nucleon scattering as a function of collision energy (experimental data are taken from Particle Physics Data System http://wwwppds. ihep.su:8001/ppds.html).

Figure 5 shows the approximate Regge trajectories of the "soft" Pomeron $\alpha_P(t)$ and the secondary Reggeons $\alpha_+(t)$ and $\alpha_-(t)$ obtained as a result of parameter fitting. Note that although the intersections of $\alpha_+(0)$ and $\alpha_-(0)$ are higher than the intersections of the corresponding Chew–Frautchi plots, they have considerably smaller slopes at $t = 0$ (see Table 1). Therefore, the smooth and monotonic matching of Chew–Frautchi plots in the resonance region with our approximations to meson trajectories in the scattering region is not impossible, and thus, the presented quantitative estimates for secondary trajectories in the region $0\ \mathrm{GeV}^2 < -t < 2\ \mathrm{GeV}^2$ do not contradict asymptotic perturbative relations (11), phenomenology, and relations (14).

For comparison, Table 3 gives the quality of data description obtained by other authors for phenomenological description of the same processes.

We especially mention [25], in which the best phenomenological description of data in a wide kinematic region is presented. In that work, the so called "dipole" Pomeron model is used, where the leading term in the scattering amplitude has the form

$$a_d(s, t) \sim \frac{\partial}{\partial j}\left[\frac{\beta_d(j, t)}{\sin\frac{\pi j}{2}}\left(-i\frac{s}{s_0}\right)^{j-1}\right]\Bigg|_{j = \alpha_d(t)}$$

$$= \left[\frac{\partial \ln \beta_d(j, t)}{\partial j}\Bigg|_{j = \alpha_d(t)} + \frac{\pi}{2}\tan\frac{\pi(\alpha_d(t) - 1)}{2} + \ln\left(-i\frac{s}{s_0}\right)\right]$$

$$\times \frac{\beta_d(\alpha_d(t), t)}{\sin\frac{\pi\alpha_d(t)}{2}}\left(-i\frac{s}{s_0}\right)^{\alpha_d(t)-1},$$

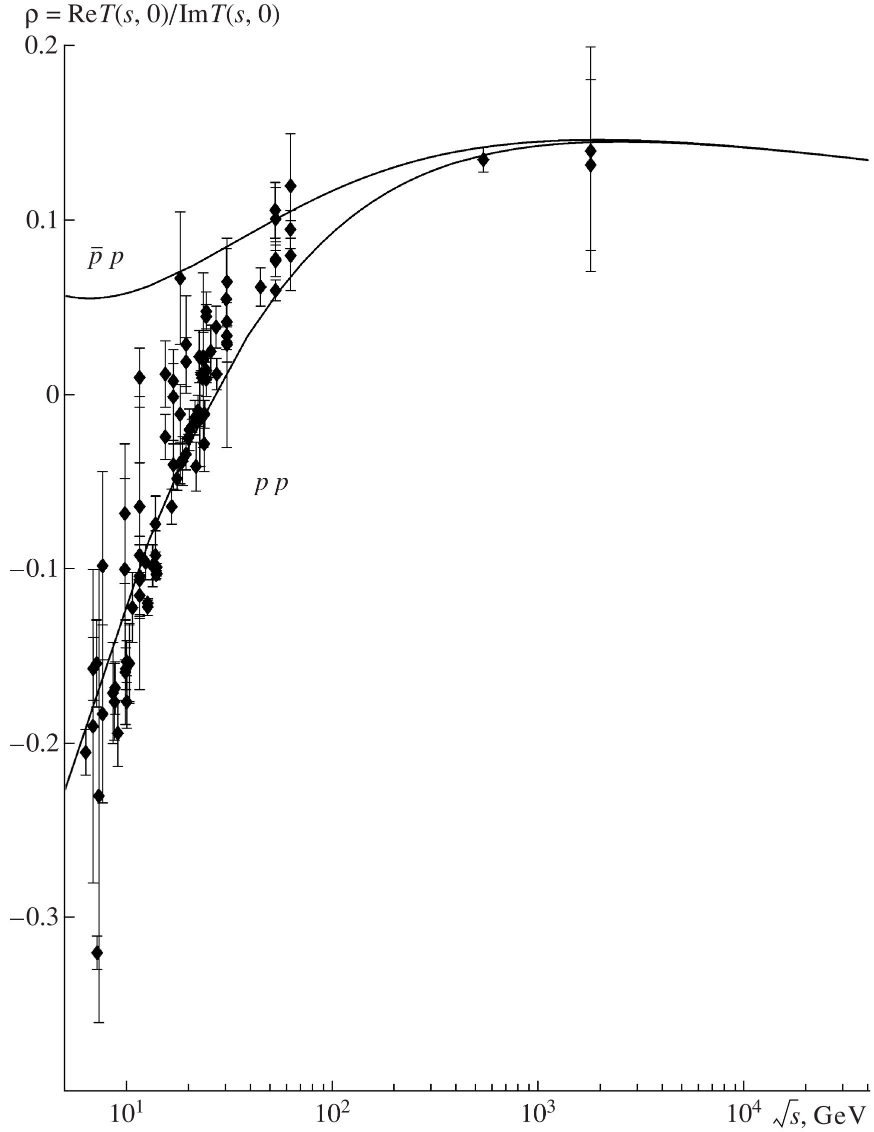

Fig. 4. Ratio of the real and imaginary parts of the amplitude of nucleon–nucleon forward scattering as a function of collision energy (experimental data are taken from Particle Physics Data System http://wwwppds. ihep.su:8001/ppds.html).

where $s_0 = 1$ GeV^2, $\alpha_d(t) = 1 + \alpha'_d t$, and $\alpha'_d \approx 0.31$ GeV^{-2}.

Unfortunately, upon description of experimental data, in [25], the first and second terms in square brackets are neglected; in the general case, this is not justified in the region $\sqrt{s} < 50$ GeV, $-t > 1$ GeV^2 when $\left|\frac{\pi}{2}\tan\frac{\pi(\alpha_d(t)-1)}{2}\right| > 0.8$ and $\ln\left(\frac{s}{s_0}\right) < 8$. To avoid this difficulty upon description of experimental data in the kinematic region $\sqrt{s} > 5$ GeV, 0.1 GeV$^2 < -t < 6$ GeV^2, it is necessary to impose a specific constraint on the behavior of the unknown analytic function $\beta_d(j, t)$,

$$\left|\frac{\partial}{\partial j}\left[\ln\frac{\beta_d(j,t)}{\sin\frac{\pi j}{2}}\right]\right|_{j = 1+\alpha'_d t} \ll \ln\left(\frac{s}{s_0}\right),$$

$$(\sqrt{s} > 5 \text{ GeV}, 0.1 \text{ GeV}^2 < -t < 6 \text{ GeV}^2),$$

(the sign "≪" in the last inequality means that it is possible to neglect the corresponding term in the amplitude without a considerable change in χ^2/d.o.f.).

This constraint, which is not mentioned in [25] but is explicitly present in the "dipole" Pomeron model, has no substantiation from QCD or general principles, and in our opinion, it deprives the phenomenological scheme presented in [25] of its theoretical basis (the

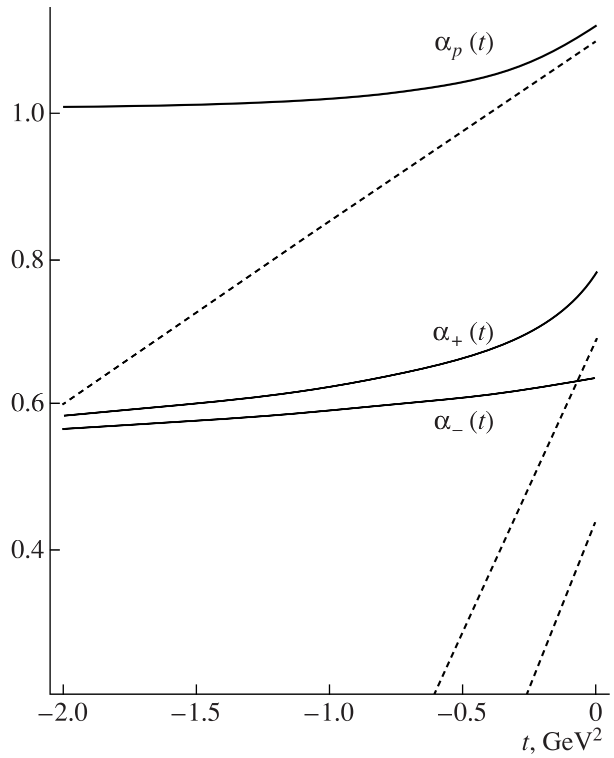

Fig. 5. Approximate Regge trajectories of the "soft" Pomeron and the Reggeons f_2/a_2 and ω/ρ obtained by fitting free parameters; dashed lines $\alpha_f^{\rm lin}(t) = 0.69 + 0.81t$ and $\alpha_\omega^{\rm lin}(t) = 0.44 + 0.92t$ are the continuations of Chew–Frautchi plots for the Reggeons f_2 and ω, and $\alpha_P^{\rm lin}(t) = 1.1 + 0.25t$ is the linear trajectory of the "soft" Pomeron commonly used in the literature.

same reasoning can be applied to the "triple" Pomeron model also considered in [25]).

CONCLUSIONS

Let us summarize what was said above. On the example of the elastic diffractive proton–(anti)proton scattering, it was shown that the explicit account of asymptotic properties of Regge trajectories in the perturbative region provides the possibility of not only avoiding difficulties related to the occurrence of nonphysical signature singularities in the scattering region, but qualitatively reproducing the diffraction pattern in a sufficiently wide kinematic region in the framework of the minimal phenomenological scheme with a clear physical meaning, taking into account only those Regge trajectories whose considerable contributions cannot be doubted. This is the main practical advantage of the proposed approach, as compared to the application of linear Regge trajectories, for which a more or less satisfactory description of experimental data is possible only if a larger number of Reggeons is used [20–22, 25].

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