

Nonlinearity of Regge trajectories in the scattering region

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ABSTRACT: The nonlinearity of Regge trajectories at real negative values of the argument is discussed as their general QCD-inspired property. The processes of elastic diffractive scattering $p+p \rightarrow p+p$ and $\bar{p}+p \rightarrow \bar{p}+p$ at collision energies $\sqrt{s} > 23 \text{ GeV}$ and transferred momenta squared $0.005 \text{ GeV}^2 < -t < 3 \text{ GeV}^2$ are considered in the framework of the Regge-eikonal model [3]. By comparison of phenomenological estimates with available experimental data on angular distributions it is demonstrated that in this kinematical range the data can be satisfactorily described as if taking into account only three nonlinear Regge trajectories with vacuum quantum numbers (“soft” pomeron, C -even f_2/a_2 -reggeon and C -odd ω/ρ -reggeon). It is also shown that their nonlinearity is essential and not to be ignored. The correspondence of the Kwiecinski $q\bar{q}$ -pole [26] to the secondary reggeons and the relevance of the Kirschner-Lipatov “hard” pomeron pole [25] to elastic diffraction are discussed.

KEYWORDS: Phenomenological Models, Nonperturbative Effects.

Contents

1. Introduction	1
2. The QCD pomeron	4
3. The model	6
4. The fitting results	8
5. Conclusions	12

1. Introduction

Our goal is to substantiate that nonlinearity of leading Regge trajectories at negative values of the argument (nonlinearity of trajectories in the resonance energy region is discussed in [1, 2]) following from some general requirements is essential and not to be neglected under considering strong interaction phenomena in the framework of Regge-based models. We will demonstrate this on the example of high energy elastic (anti)pp-scattering and will exploit for this purpose the Regge-eikonal model [3]. We do not intend to compete with [4–11] and other authors in getting the lowest value of $\chi^2/\text{d.o.f.}$ over all available data on angular distributions at high energies.

The nonlinearity of the leading trajectory was demonstrated in the experiment on measurement of single diffraction cross-sections [12]. Also, the use of the effective nonlinear dipole pomeron trajectory provided the successful description of existing data on high-energy elastic (anti)pp-scattering [5], on photoproduction of vector mesons [13, 14] and deeply virtual Compton scattering [15].

Note also that authors [4, 16] as well as many others insist on the linearity of some leading Regge trajectories in the Euclidean domain at least at $-2\text{GeV}^2 < t < 0$. The first phenomenological argument is a natural desire to continue Chew-Frautschi plots to the region of negative values of the argument. But all poles corresponding to resonances are situated on different unphysical sheets since they possess not only the mass but also the nonzero width. Hence, appropriate points on any Chew-Frautschi plot pertain to different branches of the corresponding analytic function and straight continuation to the region below the lowest threshold is not correct from the analytical point of view (although it can be used, for example, for rough estimation of the intercept value). We have to conclude that hadron spectroscopy does not provide absolutely reliable grounds for determining the behavior of Regge trajectories in the Euclidean domain. The second “evidence” of the linearity originates from the data on exchange processes $\pi^- + p \rightarrow \pi^0 + n$ and $\pi^- + p \rightarrow \eta + n$ in

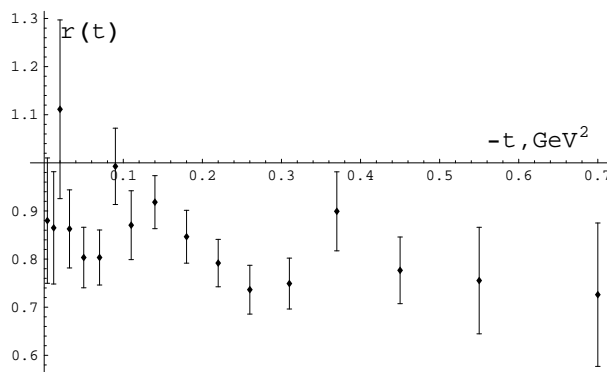


Figure 1: Function $r(t)$ extracted from the data on exchange processes $\pi^- + p \rightarrow \pi^0 + n$ and $\pi^- + p \rightarrow \eta + n$ at collision energies 11.0 and 19.4 GeV in the Born approximation.

the Born approximation [17]. Since within this approach only two of the leading reggeons (ρ and a_2) give contribution to the amplitude and we can consider these trajectories approximately equal (as a consequence of the weak degeneracy following from the hadron spectroscopy data) one could try to extract them directly from the corresponding high-energy angular distributions [18]. The appropriate formula is

$$\alpha(t) = 1 + \frac{1}{2} \left(\ln \frac{s_1}{s_2} \right)^{-1} \left(\ln \frac{d\sigma}{dt} \Big|_{s=s_1} - \ln \frac{d\sigma}{dt} \Big|_{s=s_2} \right). \quad (1.1)$$

For any reaction and any different collision energies $\sqrt{s_1}$ and $\sqrt{s_2}$ the function $\alpha(t)$ must be the same (if the Born approximation works) and, hence, the function $r(t) \equiv (1 - \alpha^{\pi^0+n}(t))/(1 - \alpha^{\eta+n}(t))$ (where $\alpha^{\pi^0+n}(t)$ and $\alpha^{\eta+n}(t)$ are the trajectories from angular distributions for processes $\pi^- + p \rightarrow \pi^0 + n$ and $\pi^- + p \rightarrow \eta + n$ correspondingly at collision energies $\sqrt{s_1} = 11.0 \text{ GeV}$ and $\sqrt{s_2} = 19.4 \text{ GeV}$) must be strictly equal to unity at any argument value. However, this is not evident after extraction this function from the data on exchange reactions (figure 1). The only conclusion which an unprejudiced person can draw looking at this picture is that the Born approximation seems invalid at transferred momenta $-t > 0.2 \text{ GeV}^2$ and, consequently, the linearity of the ρ/a_2 -trajectory at higher scattering angles is not guaranteed.

So, linear parametrizations for Regge trajectories have no phenomenological advantages over nonlinear ones. Also note that the best fit to the trajectory extracted from the earlier data on $\pi^- + p \rightarrow \eta + n$ [19] under exploiting the Born approximation was obtained for nonlinear parametrization [20, 21]

$$\alpha(t) = \alpha(\infty) + \frac{[\alpha(0) - \alpha(\infty)]^2}{\alpha(0) - \alpha(\infty) - t\alpha'(0)} \quad (1.2)$$

with $\alpha(\infty) = 0$.

Our viewpoint is based on the conviction that QCD is the fundamental theory of strong interaction. This rather general requirement imposes restrictions on the behavior of Regge trajectories in the range of the perturbative QCD validity since at large transferred

momenta exchanges by single reggeons must turn into exchanges by colour-singlet parton combinations. If we assume that the reggeon exchange giving the leading contribution to the eikonal at high energies (pomeron) turns into some multi-gluon exchange in the perturbative range we will come to [22–25]

$$\lim_{t \rightarrow -\infty} \alpha_{gg\dots g}(t) = 1. \tag{1.3}$$

In the case of the quark-antiquark pair (f_2 -reggeon, ω -reggeon etc.) one obtains [26]

$$\alpha_{\bar{q}q}(t) = \sqrt{\frac{8}{3\pi} \alpha_s(\sqrt{-t})} + o(\alpha_s^{1/2}(\sqrt{-t})) \tag{1.4}$$

where $\alpha_s(\mu) \equiv g_s^2(\mu)/4\pi$ is the running coupling.¹

This property of Regge poles (tending to constant at $t \rightarrow -\infty$) is quite general and follows from the fact of their invariance relative to the renormalization group transformations (the requirement of renorm-invariance is well-grounded by observability of bound states and resonances). The general solution of the renorm-group differential equation for the renorm-invariant quantity $f(\frac{t}{\mu^2}, \alpha_s(\mu))$ is (in the case of massless fields) of the form [28]

$$f\left(\frac{t}{\mu^2}, \alpha_s(\mu)\right) = \Phi\left(\frac{t}{\mu^2} e^{K(\alpha_s(\mu))}\right) \tag{1.5}$$

where $\Phi(x)$ is an arbitrary function analytic in the region defined by the analyticity in t , μ is the renormalization scale, $K'(\alpha_s) = 1/\beta(\alpha_s) \equiv (\mu^2 \frac{\partial \alpha_s(\mu)}{\partial \mu^2})^{-1}$. For any quantum field model with asymptotic freedom (1.5) leads in the perturbative sector ($\mu = \sqrt{-t}$, $t \rightarrow -\infty$) to

$$\lim_{t \rightarrow -\infty} f(t) = const. \tag{1.6}$$

Actually, this corresponds to the existence of the free field limit as $\alpha_s(\sqrt{-t}) \rightarrow 0$. In the theory of potential scattering the squared effective radius of interaction corresponding to the reggeon exchange [20, 29] $R^2 \sim \alpha'(t)(2\alpha(t) + 1) \rightarrow 0$ at $t \rightarrow -\infty$, $\alpha(t) \rightarrow const$. This purely quantum mechanical result is in agreement with the property of asymptotic freedom at short distances (in the case of linear trajectory $\alpha(t)$ ($\alpha'(t) > 0$) $R^2 \rightarrow -\infty$ at $t \rightarrow -\infty$).

As a consequence of (1.6) the essential nonlinearity of the Regge trajectories takes place in the range $-\infty < t < 0$. This nonlinearity is their fundamental property.

Besides, we assume that $Im \alpha(t + i0) \geq 0$ increases slowly enough at $t \rightarrow +\infty$ (for example, not faster than $Ct \ln^{-1-\epsilon} t$, $\epsilon > 0$) so that the dispersion relations with not more than one subtraction take place, i.e.

$$\alpha(t) = \alpha_0 + \frac{t}{\pi} \int_{t_T}^{+\infty} \frac{Im \alpha(t' + i0)}{t'(t' - t)} dt', \tag{1.7}$$

¹The universality of the asymptotic vanishing at $t = -\infty$ of meson trajectories seems to contradict the existence of (pseudo)scalars. In fact, if $\alpha_\pi(t)$ behaved like (1.4) at $t \rightarrow -\infty$ it could not be monotonic function of t . Possible way to preserve the monotony is to suppose that at $t \rightarrow -\infty$ the trajectory tends to some negative value. Similar behavior takes place, e.g., in the phenomenological studies of the trajectories containing heavy quarkonia [27]. To what extent is it possible in QCD remains unclear.

and $Im \alpha(t + i0) \geq 0$ at $t \geq t_T > 0$ (we would like to point out that these assumptions are strictly fulfilled in the theory of perturbations and the theory of potential scattering [30]). In this case we obtain [20, 30]

$$\frac{d^n \alpha(t)}{dt^n} > 0 \quad (t < t_T, n = 1, 2, 3, \dots). \quad (1.8)$$

The t -channel unitarity implies some special t -dependence of the trajectories near the threshold [30]

$$Im \alpha(t) \sim (t - t_T)^{\alpha(t_T)+1/2} \quad (t \geq t_T).$$

As was argued in [31] this must have observable effects. However, in this paper we do not take this requirement into account, considering such effects as a kind of “fine structure” which is beyond the accuracy level we adopted.

It is shown in [32] that the assumption of the linearity of Regge trajectories leads to the situation when the diffractive pattern at high energy (anti)proton-proton elastic scattering can be described in the framework of the Regge-eikonal approach only after introduction of several vacuum reggeons giving contributions to the eikonal (one needs not less than three C -even poles with intercepts higher than unity). In other Regge-based models one also need more than one reggeon with intercept higher than unity (see, for example, [5, 6, 11]). The use of nonlinear trajectories allows us to curtail the number of reggeons essential for an acceptable description of the data on high energy elastic nucleon-nucleon scattering. Namely, using only one trajectory with vacuum quantum numbers and intercept higher than unity (“soft” pomeron) and two secondary reggeons with intercepts lower than unity (C -even f_2/a_2 -reggeon and C -odd ω/ρ -reggeon) we will be able to describe available experimental data in a wide kinematical range $\sqrt{s} > 23 GeV$, $0.005 GeV^2 < -t < 3 GeV^2$ (at $-t > 3 GeV^2$ we can not ignore the contributions from other leading vacuum reggeons — C -even “hard” pomeron(s) and C -odd odderon(s)). The very fact that we have managed to satisfactorily reproduce the diffractive pattern within this rather wide kinematical range in the framework of such a simple phenomenological scheme is quite encouraging and points to the agreement between general QCD-related theoretical conclusions and experiment.

2. The QCD pomeron

Regge trajectories which are tightly related to the hadron spectroscopy deal, generally, with confinement of quarks and gluons. Thus, without a great progress in the solution of this outstanding problem the QCD theory of Regge trajectories is still in its primordial stage.

It is natural that in the absence of a regular non-perturbative technique some progress is limited by perturbative calculations. In the literature the wide attention is being paid to so-called “BFKL pomeron” (or other “BFKL reggeons”) [33]. In this approach one strives to formulate some kind of Bethe-Salpeter equation for the gluon-gluon scattering amplitude (Green function) in which “ t -channel” gluons are actually gluon Regge trajectories (“reggeized gluons”) that have to be preliminary calculated from another Bethe-Salpeter

equation for the colour-octet t -channel. This trick can make an impression that, say, the pomeron trajectory is not just a 2-gluon exchange (with account of interaction between these two gluons via “ladder rung exchanges”) but contains many-gluon configurations in the t -channel as well. However, the very “reggeized gluons” are related to no more than two gluons each (since they result from the the BS-type equation with 2-gluon irreducible kernels). That is one deals with no more than 4 usual gluons in the t -channel.

The use of fixed number of gluon exchanges seems to be justified at $|t|$ large enough, where the elementary short-distance structure of reggeons has to show up. As to small values of t it is clear that due to genuinely strong interaction one cannot limit the problem by any fixed number of exchanged partons. Certainly, one can argue about “valence” gluons but these have to be essentially non-perturbative and different from the “slightly reggeized” gluons mentioned above. It is even quite probable that the corpuscular language in this quasi-classical region ceases to be adequate and undulatory gluonic fields are more relevant.

In spite of such a little bit gloomy landscape one still can try to account (when describing the scattering data in terms of Regge exchanges) for such a distinctive QCD prediction as asymptotic constancy of Regge trajectories in deeply Euclidean region, $-t \rightarrow \infty$ (in other words, the nonlinearity of the pomeron and secondary reggeon trajectories which was already mentioned in the previous section). Via solving the Bethe-Salpeter-like equation for the gluon-gluon scattering amplitude it was obtained in [25] that

$$\alpha_P^{\text{hard}}(t) = 1 + \frac{12 \ln 2}{\pi} \alpha_s(\sqrt{-t}) \left[1 - \alpha_s^{2/3}(\sqrt{-t}) \left(\frac{7\zeta(3)}{2 \ln 2} \right)^{1/3} \left(\frac{3/4}{11 - 2/3 n_f} \right)^{2/3} \right], \quad t \rightarrow -\infty. \quad (2.1)$$

The second term in brackets is approximately equal to 0.09 at $t = -M_Z^2$ when $\alpha_s(M_Z) \approx 0.118$ (here $M_Z \approx 91.2 \text{ GeV}$ is the Z-boson mass and $n_f = 5$) and such an expansion is justified at this scale (at $t = -(2.5 \text{ GeV})^2$, $\alpha_s(2.5 \text{ GeV}) \approx 0.3$ this term is about 0.17). Even at $t = -M_Z^2$ the value of $\alpha_P^{\text{hard}}(-M_Z^2) \approx 1.28$ is quite high ($\alpha_P^{\text{hard}}(-6 \text{ GeV}^2) \approx 1.66$). If one assumes the monotony of $\alpha_P^{\text{hard}}(t)$ the intercept, $\alpha_P^{\text{hard}}(0)$, has to lie even higher. The last statement agrees with a rough estimate of the lower bound for the BFKL pomeron intercept value $\alpha_P^{\text{hard}}(0) - 1 \geq 0.3$ obtained in [34]. Such a value can be hardly relevant for the data on elastic diffraction.

Some time ago a NLO result was obtained for the pomeron intercept [35]

$$\alpha_P(0) = 1 + \frac{12 \ln 2}{\pi} \alpha_s(\mu) \left(1 - \frac{20}{\pi} \alpha_s(\mu) \right). \quad (2.2)$$

This expression cannot be accepted as a true value of the pomeron intercept as it depends on the renormalization scheme and arbitrary renormalization scale μ via α_s . As was argued recently [36] within the limit of massless quark fields the true intercept of any Regge trajectory has to be strictly independent of the QCD coupling constant.

These circumstances enforce us (while waiting for further theoretical progress) to try some purely phenomenological ansatz for the pomeron trajectory which retains but one feature of (2.1): it tends to 1 at $|t|$ high enough (see relation (1.3)).

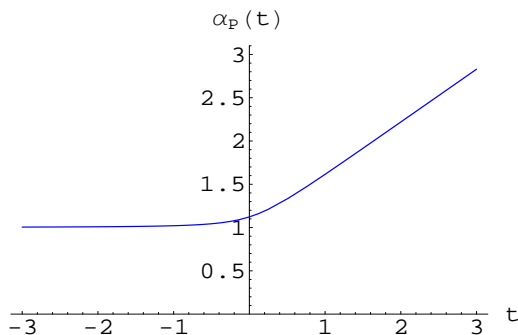


Figure 2: The qualitative behavior of function $\alpha_P(t)$ from (3.2).

3. The model

Since our primary goal is not to get the best phenomenological description of all available data on elastic diffraction at high energies but to find a phenomenological confirmation of such a macroscopic phenomenon as following from QCD nonlinear behavior of Regge trajectories in the scattering region we choose the eikonal to have the simplest form (ignoring the contribution of “hard” pomeron(s), odderon(s) etc.)

$$\delta(s, t) = \delta_P(s, t) + \delta_+(s, t) \mp \delta_-(s, t) = \left(i + tg \frac{\pi(\alpha_P(t) - 1)}{2} \right) \beta_P(t) \left(\frac{s}{s_0} \right)^{\alpha_P(t)} + \quad (3.1)$$

$$+ \left(i + tg \frac{\pi(\alpha_+(t) - 1)}{2} \right) \beta_+(t) \left(\frac{s}{s_0} \right)^{\alpha_+(t)} \mp \left(i - ctg \frac{\pi(\alpha_-(t) - 1)}{2} \right) \beta_-(t) \left(\frac{s}{s_0} \right)^{\alpha_-(t)}$$

where $s_0 \equiv 1 \text{ GeV}^2$, $\alpha_P(t)$ and $\beta_P(t)$ are the trajectory and the residue of the “soft” pomeron, $\alpha_+(t)$ and $\alpha_-(t)$ are the trajectories of secondary reggeons (i.e. we assume that $\alpha_+(t) \approx \alpha_{f_2}(t) \approx \alpha_{a_2}(t)$ and $\alpha_-(t) \approx \alpha_\omega(t) \approx \alpha_\rho(t)$ due to the isospin symmetry of the quark flavors), $\beta_+(t) \equiv \beta_{f_2}(t) + \beta_{a_2}(t)$ and $\beta_-(t) \equiv \beta_\omega(t) + \beta_\rho(t)$, the sign “-” (“+”) before $\delta_-(t)$ corresponds to the particle-on-particle (particle-on-antiparticle) scattering.

Our phenomenological ansatz for the “soft” pomeron pole function is (see figure 2)

$$\alpha_P(t) = 1 + p_1 \left[1 - p_2 t \left(\text{arctg}(p_3 - p_2 t) - \frac{\pi}{2} \right) \right]. \quad (3.2)$$

It bears the above-mentioned characteristic feature, i.e. tends to 1 at $t \rightarrow -\infty$. At large and positive t (3.2) mimics the “stringy” behavior, i.e. grows linearly with t . We do not give a great significance to this, though. Moreover, such a behavior, if taken seriously, leads to complex singularities of $\alpha_P(t)$ and one has to provide a special care to save the amplitude from causality (analyticity) violations [37].²

²Earlier, in [38] there was proposed a similar form for the pomeron trajectory $\alpha_P(t) = 1 + \alpha'_P(0) s_1 \text{arctg} \frac{t}{s_1}$. This function also flattens at $t \rightarrow -\infty$ but in [38] s_1 was chosen large enough for the linear approximation was valid in the diffraction region. As a consequence, $\lim_{t \rightarrow -\infty} \alpha_P(t) = 1 - \alpha'_P(0) s_1 \frac{\pi}{2} < 0$. So, our approach (3.2) is quite different because (as it will be shown below) asymptotic relation (1.3) together with the requirement of the monotony of $\alpha_P(t)$ results in a strong nonlinearity of the pomeron trajectory.

Residues are chosen as

$$\beta_P(t) = B_P e^{b_P t} (1 + d_1 t + d_2 t^2 + d_3 t^3 + d_4 t^4), \quad \beta_+(t) = B_+ e^{b_+ t}, \quad \beta_-(t) = B_- e^{b_- t}. \quad (3.3)$$

Our parametrization for the secondary trajectories contains the QCD-inspired expressions

$$\alpha_+(t) = \left(\frac{8}{3\pi} \gamma(\sqrt{-t + c_+}) \right)^{1/2}, \quad \alpha_-(t) = \left(\frac{8}{3\pi} \gamma(\sqrt{-t + c_-}) \right)^{1/2} \quad (3.4)$$

where

$$\gamma(\mu) \equiv \frac{4\pi}{11 - \frac{2}{3}n_f} \left(\frac{1}{\ln \frac{\mu^2}{\Lambda^2}} + \frac{1}{1 - \frac{\mu^2}{\Lambda^2}} \right) \quad (3.5)$$

is the so-called one-loop analytical QCD effective coupling constant [39], $n_f = 3$ is the number of quark flavors taken into account, $\Lambda = \Lambda^{(3)} = 0.346 \text{ GeV}$ is the QCD dimensional parameter (the value was taken from [40]) and $c_+, c_- > 0$ are free phenomenological parameters.

This analytic approximation for the secondary reggeons is obtained in the following way. We take an expression for the $q\bar{q}$ Regge pole in the perturbative sector [26] derived via solving the Bethe-Salpeter-like equation in the range of the perturbative QCD validity and then replace the perturbative effective coupling constant to expression (3.5) obtained in the framework of the dispersive approach [39]. Further, the introduction of the free parameters c_+ and c_- is the simplest phenomenological way to take into account the disparity between $\alpha_+(t)$ and $\alpha_-(t)$ and not to spoil the asymptotic behavior of these trajectories in the perturbative sector ($-t \gg 1 \text{ GeV}^2$).

To obtain angular distributions we substitute (3.2), (3.3), (3.4) into (3.1), proceed via Fourier-Bessel transformation

$$\delta(s, b) = \frac{1}{16\pi s} \int_0^\infty d(-t) J_0(b\sqrt{-t}) \delta(s, t) \quad (3.6)$$

to the coordinate representation, using the eikonal representation of the scattering amplitude

$$T(s, b) = \frac{e^{2i\delta(s, b)} - 1}{2i} \quad (3.7)$$

through inverse Fourier-Bessel transformation

$$T(s, t) = 4\pi s \int_0^\infty db^2 J_0(b\sqrt{-t}) T(s, b) \quad (3.8)$$

obtain its value in the momentum representation (during numerical calculating integrals from (3.6), (3.8) we change upper limits of integration to 4 GeV^2 and $400 \text{ GeV}^{-2} \approx (4 \text{ Fm})^2$ correspondingly) and substitute it into the expression for the differential cross-section

$$\frac{d\sigma}{dt} = \frac{|T(s, t)|^2}{16\pi s^2}. \quad (3.9)$$

In the conclusion of this section we must note that our consideration of the high- $|t|$ behavior provides the explanation of their nonlinearity in the framework of QCD but we

	pomeron		f_2/a_2-reggeon		ω/ρ-reggeon
p_1	0.123	c_+	0.1 GeV^2	c_-	0.9 GeV^2
p_2	1.58 GeV^{-2}				
p_3	0.15				
B_P	43.5	B_+	153	B_-	46
b_P	2.4 GeV^{-2}	b_+	4.7 GeV^{-2}	b_-	5.6 GeV^{-2}
d_1	0.43 GeV^{-2}				
d_2	0.39 GeV^{-4}				
d_3	0.051 GeV^{-6}				
d_4	0.035 GeV^{-8}				
$\alpha_P(0)$	1.123	$\alpha_+(0)$	0.78	$\alpha_-(0)$	0.64
$\alpha'_P(0)$	0.28 GeV^{-2}	$\alpha'_+(0)$	0.63 GeV^{-2}	$\alpha'_-(0)$	0.07 GeV^{-2}

Table 1: Parameters obtained by fitting to the data.

do not intend to say that our approach is capable to give the description of cross-sections at fixed angles. Our concrete parametrizations for the unknown functions $\alpha_P(t)$, $\alpha_+(t)$, $\alpha_-(t)$, $\beta_P(t)$, $\beta_+(t)$, $\beta_-(t)$ in the eikonal (3.1) are purely phenomenological quantitative approximations valid in the soft diffraction region (only relations (1.3), (1.4), (1.8) have physical sense). Nonetheless, the QCD asymptotics of the trajectories (tending to constants at $t \rightarrow -\infty$) are fairly compatible with, e.g. “quark counting rules” [24]. The dual amplitude with Mandelstam analyticity model (DAMA) with logarithmic trajectories [41] provides an alternative way to reproduce the QCD-type behavior at fixed angles.

4. The fitting results

Turn to the description of experimental data. The results of fitting over data on angular distributions in the kinematical region $\sqrt{s} > 23 \text{ GeV}$, $0.005 \text{ GeV}^2 < -t < 3 \text{ GeV}^2$ [42]³ are represented in table 1, 2 and figure 3, 4.

If we compare angular distributions obtained using parametrization (3.2), (3.3), (3.4) with those obtained using the same values of free parameters but with replacement of the nonlinear pomeron trajectory to its linear approximation (only two first terms in the Taylor expansion) we will disclose a huge difference between the corresponding results (see figure 3, 4) which is a consequence of the fact that the simultaneous fulfilment of the conditions $\alpha'_P(t) > 0$ at $t \leq 0$, $\lim_{t \rightarrow -\infty} \alpha_P(t) = 1$ together with the phenomenological estimates $\alpha'_P(0) > 0.2 \text{ GeV}^{-2}$, $\alpha_P(0) < 0.15$ makes the approximation $\alpha_P(t) = \alpha_P(0) + \alpha'_P(0)t$ for the pomeron trajectory invalid in the region $-t > 0.8 \text{ GeV}^2$, i.e. the effect of the nonlinearity turns out strong even at low $-t$ (see figure 5, 6). Here we must emphasize that the linear approximations to our nonlinear trajectories differ from linear trajectories used by other authors. In the most of papers quoted in table 3 linear Regge trajectories were

³For calculation of electromagnetic corrections to the scattering amplitude we used the recipe by R. Cahn [43] (see also [44]).

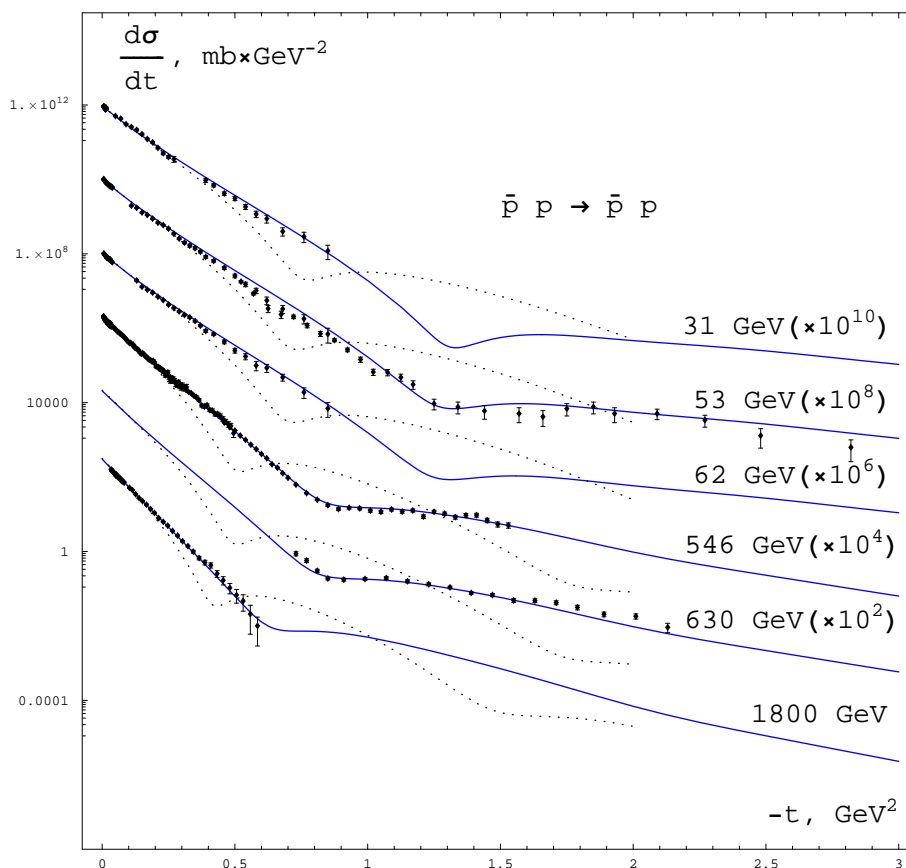


Figure 3: Differential $\bar{p} p \rightarrow \bar{p} p$ cross-sections for the cases of nonlinear (solid lines) and linearly approximated (dotted lines) “soft” pomeron trajectory.

successfully applied to the phenomenological description of the data but QCD asymptotic relations (1.3), (1.4) were ignored.

In figure 7 the predictions for the total cross-section dependence on the center-of-mass energy are shown. In particular, $\sigma_{\text{tot}}(200 \text{ GeV}) \approx 52 \text{ mb}$, $\sigma_{\text{tot}}(14 \text{ TeV}) \approx 111 \text{ mb}$. The noticeable disagreement with the experimental $\bar{p} p$ data at $\sqrt{s} < 13 \text{ GeV}$ points to the fact that at such energies the simplest phenomenological scheme (3.1) is inapplicable for satisfactory quantitative description of the data and the contribution of f'_2 - and ϕ -reggeons must be taken into account. Since the corresponding terms in the imaginary part of the eikonal have equal (opposite) signs for the $\bar{p} p$ (pp) scattering the discrepancy between the model curve and the $\bar{p} p$ data turns out larger than for the pp data. Also, in this figure by comparison of the imaginary part of the forward amplitude with the same from the Born approximation it is demonstrated that we can not neglect absorptive corrections which significantly reduce the amplitude value. Figure 8 represents the dependence of ρ -parameter on the center-of-mass energy.

In the last two figures the “soft” pomeron and secondary trajectories and their slopes as functions of the transferred momentum squared are presented. Point out that although

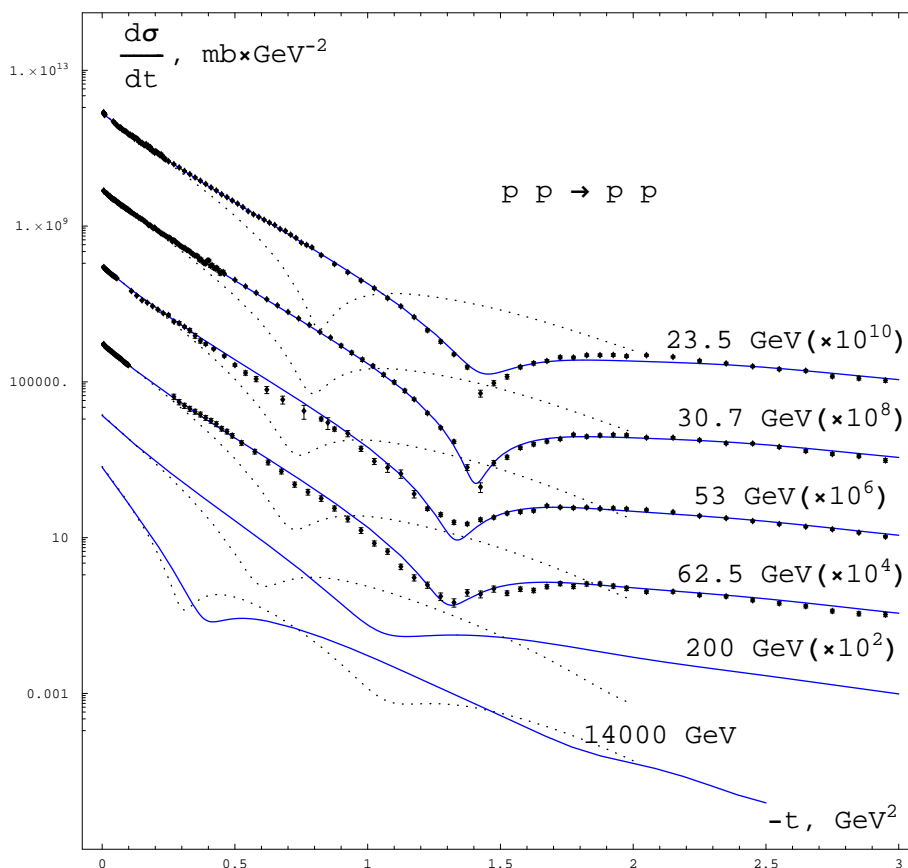


Figure 4: Differential $pp \rightarrow pp$ cross-sections for the cases of nonlinear (solid lines) and linearly approximated (dotted lines) “soft” pomeron trajectory.

Set of data	Number of points	χ^2
$\sqrt{s} = 23 \text{ GeV} (pp)$	124	280
$\sqrt{s} = 31 \text{ GeV} (pp)$	154	467
$\sqrt{s} = 53 \text{ GeV} (pp)$	85	423
$\sqrt{s} = 62 \text{ GeV} (pp)$	107	409
$\sqrt{s} = 31 \text{ GeV} (\bar{p}p)$	38	108
$\sqrt{s} = 53 \text{ GeV} (\bar{p}p)$	60	336
$\sqrt{s} = 62 \text{ GeV} (\bar{p}p)$	40	156
$\sqrt{s} = 546 \text{ GeV} (\bar{p}p)$	181	352
$\sqrt{s} = 630 \text{ GeV} (\bar{p}p)$	19	78
$\sqrt{s} = 1800 \text{ GeV} (\bar{p}p)$	50	129
Total	858	2738

Table 2: The quality of description of data on angular distributions.

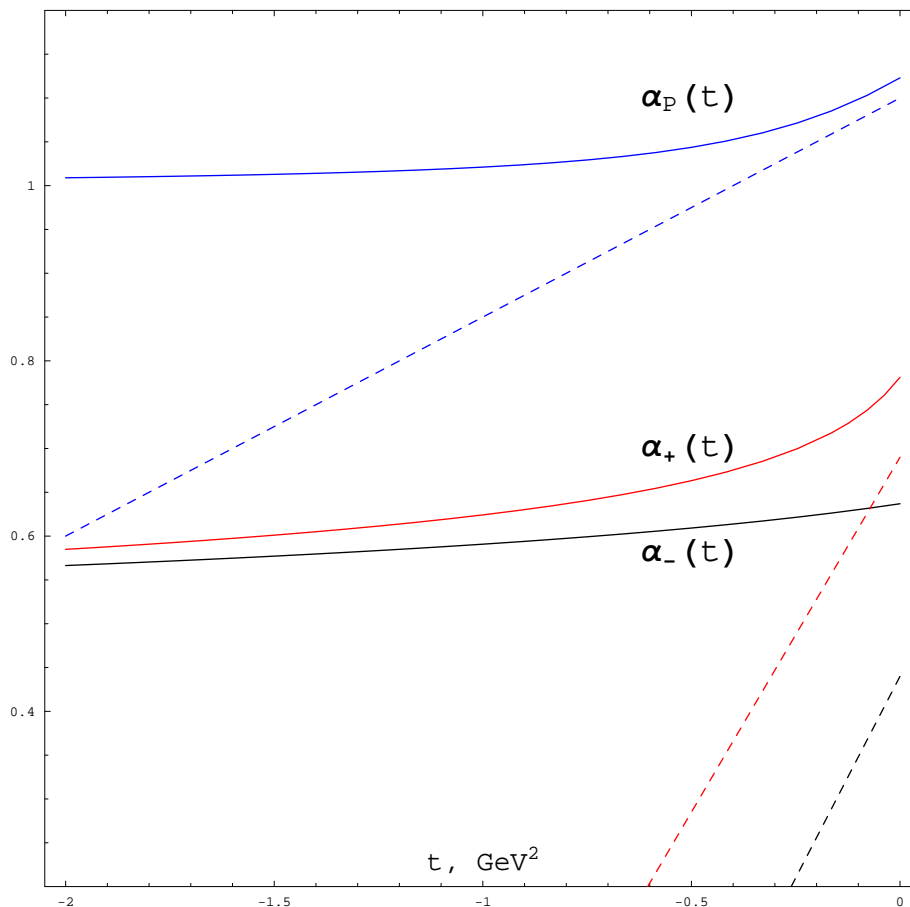


Figure 5: Approximate “soft” pomeron and secondary trajectories obtained in the fitting procedure (dashed lines: $\alpha_f^{\text{lin}}(t) = 0.69 + 0.81 t$ and $\alpha_\omega^{\text{lin}}(t) = 0.44 + 0.92 t$ are the continuations of the Chew-Frautschi plots corresponding to f_2 -reggeon and ω -reggeon and $\alpha_P^{\text{lin}}(t) = 1.1 + 0.25 t$ is the linear “soft” pomeron trajectory usually used in literature).

Ref.	$\chi^2/d.o.f.$	kinematical range
[4]	Not presented	$23 \text{ GeV} \leq \sqrt{s} \leq 546 \text{ GeV}$
[5]	2.0	$53 \text{ GeV} \leq \sqrt{s} \leq 630 \text{ GeV}, \quad 0 < -t \leq 5 \text{ GeV}^2$
[6]	2.4	$19 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}, \quad 0.1 \text{ GeV}^2 \leq -t \leq 14 \text{ GeV}^2$
[7]	Not presented	$23 \text{ GeV} \leq \sqrt{s} \leq 546 \text{ GeV}$
[8]	4.3	$23 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}, \quad 0 < -t \leq 6 \text{ GeV}^2$
[9]	Not presented	$546 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}$
[10]	2.8	$23 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}, \quad 0.01 \text{ GeV}^2 \leq -t \leq 14 \text{ GeV}^2$
[11]	1.5	$6 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}, \quad 0.1 \text{ GeV}^2 \leq -t \leq 6 \text{ GeV}^2$
[32]	2.6	$23 \text{ GeV} \leq \sqrt{s} \leq 1800 \text{ GeV}, \quad 0.01 \text{ GeV}^2 \leq -t \leq 14 \text{ GeV}^2$

Table 3: Some information on $\chi^2/d.o.f.$ over the data on elastic nucleon-nucleon scattering obtained by other authors.

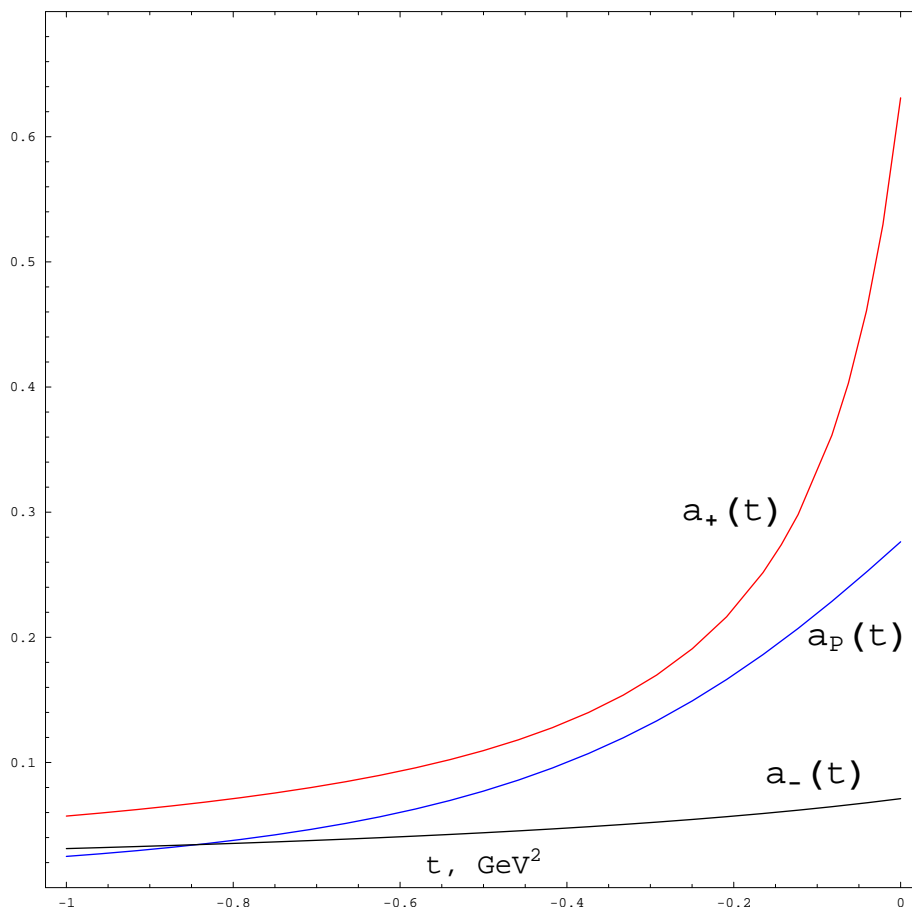


Figure 6: The slopes (in GeV^{-2}) of the Regge trajectories from the previous figure, $a_R(t) \equiv \alpha'_R(t)$.

the intercepts of secondary reggeons in our scheme are higher than those ones from the rough linear approximation based on the appropriate resonance data the corresponding slopes turn out noticeably smaller than in the linear scheme. So, our approximations (3.4) to the true secondary trajectories in the scattering region may be laced in a smooth and monotonous manner with the corresponding Chew-Frautschi plots in the resonance region without any artificial behavior at low positive values of the argument (see figure 5).

5. Conclusions

Now we can discuss the correspondence of the Kwiecinski $q\bar{q}$ -pole [26] to the secondary reggeons and the relevance of the Kirschner-Lipatov “hard” pomeron pole [25] to elastic diffraction at accessible energies and small scattering angles. It is evident that the Kwiecinski asymptotic form (1.4) continued analytically in some way to the non-perturbative sector corresponds to the phenomenological secondary Regge trajectories giving a noticeable contribution to the observed elastic diffraction cross-sections. In other words, there exists a simple analytical way to connect their asymptotic perturbative behavior (1.4) with the Regge phenomenology in the non-perturbative sector.

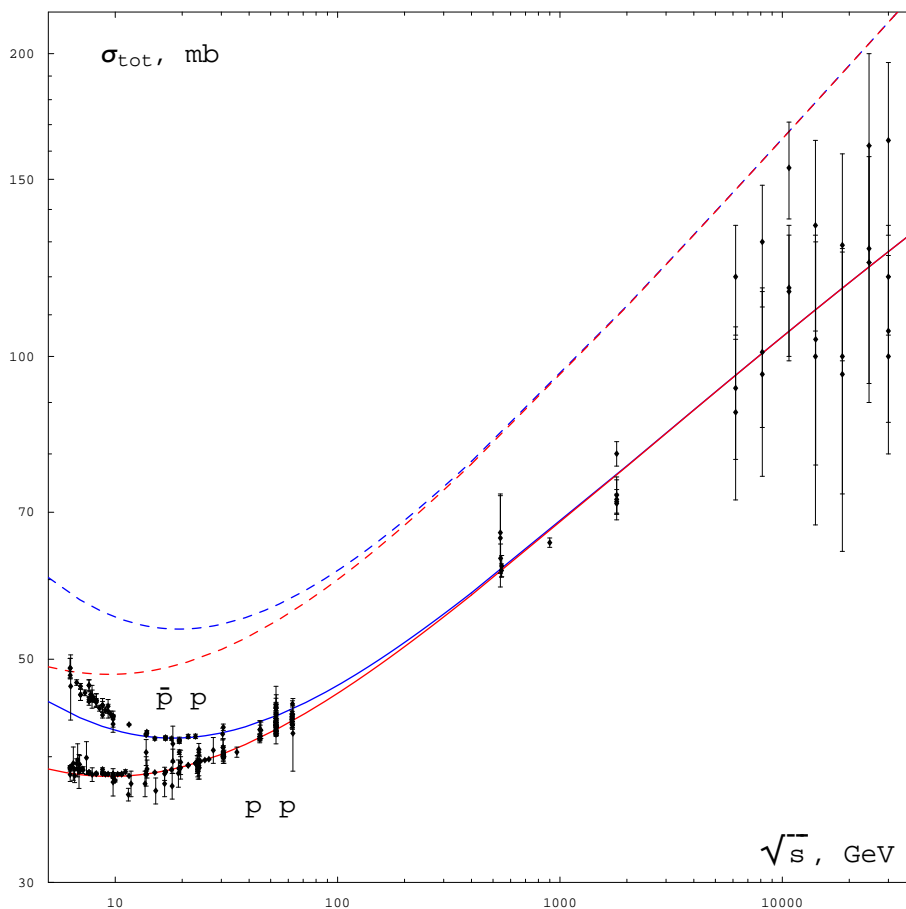


Figure 7: Total cross-sections (the eikonized amplitudes — solid lines, the Born amplitudes — dashed lines) for high-energy nucleon-nucleon scattering as functions of center-of-mass energy (experimental data were taken from Particle Physics Data System <http://wwwppds.ihep.su:8001/ppds.html>).

With the Kirschner-Lipatov “hard” pomeron the situation is quite different (see section “The QCD pomeron”). Trying to apply the Kirschner-Lipatov pole to phenomenology we have an alternative: either to insist on the correspondence between the “soft” pomeron trajectory and the Kirschner-Lipatov pole and so to accept that the trajectory is not monotonous in the Euclidean domain (in this case we come to the situation when we can not exploit its expression in form (2.1) for soft diffraction at any energies) or to assume that these poles are different ones (and so to presume that the Kirschner-Lipatov pole contribution to the eikonal is suppressed in the residue in the non-perturbative range of the argument value — it must dominate in the diffraction sector at ultra-high energies (higher than $1.8 TeV$) and also in the perturbative sector at accessible energies). The latter variant is more preferable from the analytical point of view but in both cases we are not able to use (2.1) in the Regge phenomenology of diffraction phenomena at accessible energies. As to (2.2) it gives even less reasons to use it, as explained above. The following picture seems to us reasonable. At “very high” ($-t$) we deal with pure gluon exchanges, then,

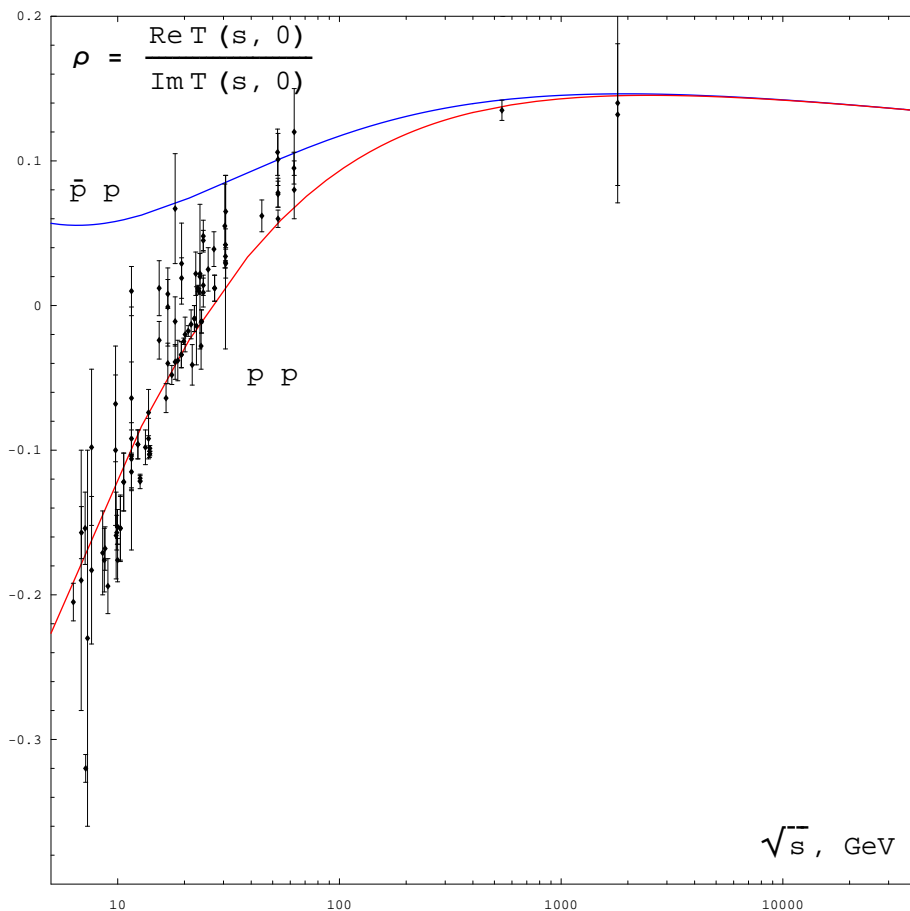


Figure 8: The forward amplitude real part to imaginary part ratio as a function of center-of-mass energy (experimental data were taken from Particle Physics Data System <http://wwwppds.ihep.su:8001/ppds.html>).

with $(-t)$ diminishing, we come to “partially collectivized” exchanges in the form of “hard pomerons” (2.1). The latter could be seen in gluon-gluon elastic scattering with colorless exchanges (“Mueller-Navelet jets”) with high $(-t)$. The hadron diffraction at $(-t) \leq 3 \text{ GeV}^2$ is dominated by “soft” or non-perturbative pomerons (reggeons) which cannot be thought as composed of definite number of partons. Unfortunately, QCD-literature has no much to say in this case.

So, in the framework of the minimal Regge-eikonal model with using those parametrizations for Regge trajectories in which their asymptotic properties and fundamental nonlinearity are taken into account

- (1) it was shown that the diffractive pattern for the elastic $\bar{p} p$ and $p p$ scattering at energies $23 \text{ GeV} < \sqrt{s} < 2 \text{ TeV}$ and scattering angles $-t < 3 \text{ GeV}^2$ is mainly formed by contribution of three Regge trajectories,
- (2) it was demonstrated (from both theoretical and phenomenological points of view) that we can not ignore the nonlinearity of the trajectories in the considered kinematical range,

- (3) the relevance of the Kwiecinski $q\bar{q}$ -pole to the secondary reggeons and the impossibility to apply the Kirschner-Lipatov trajectory to elastic diffraction at accessible energies was argued.

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